## Lissajous Art

Jules Antoine Lissajous (1822-1880) developed a technique for projecting light figures in the 1850s. He placed mirrors on tuning forks and bounced a light beam off them onto a screen. This became a standard technique for studying vibration and enabled major discoveries by Helmholtz and others.

Lissajous curves are easily produced on an oscilloscope (invented in 1897) if two sources of sine tones are available. One tone is applied to the vertical deflection and the other replaces the usual sawtooth horizontal deflection. The mode is usually labeled $\mathrm{X}-\mathrm{Y}$. Films of these figures were used as set dressing in 50's space operas and TV shows like "Outer Limits".

In the 60's John Whitney pioneered techniques for generating the images directly on film with mechanical and computer driven devices. A the same time, artists such as Lowell Cross applied the technique to powerful laser beams to develop an interactive art form that has graced planetaria and rock shows since. Now that video projectors are readily available, the same art can be produced in a much more economical format.


Figure 1.

Figure 1 is a basic Lissajous generator. Each cycle $\sim$ object is independent with controls for frequency, phase and amplitude. The output of each is sampled byt the jit.catch~ object and stuffed into a 2 plane matrix. This matrix is disassembled into X Y pairs by jit.iter, and the pairs are processed for display in a jit.LCD as shown in figure 2. Note that figure 2 also shows how a qmetro generates timing signals for the sample by jit.catch . Jit.plot can display this sort of thing more efficiently, but this drawing technique produces a better image.


Figure 2.

The window in figure 2 shows what you should see at start up. The X and Y cycle~ objects are in phase, so the buffer contains equal values in both channels. That is displayed as a diagonal line. If you adjust the phase parameter to the Y cycle $\sim$ object, various ellipses will appear.


Figure 3A. Phase 0.1



3B. Phase 0.25


3D. Phase 0.5

Phase is in fractions of a cycle in the wavetable, so 0.25 is a cosine relationship and produces a circle. Phase of 0.5 is 180 degrees and gives a line sloping right.

Returning the phase to 0.25 , set X amplitude to 0.5


Figure 4A X amplitude 0.5


4B X and Y amplitude 0.5

This results in a vertical ellipse. Setting Y amplitude to match gives another smaller circle. Later we will see how modulating phase and amplitude will add dynamic action to the basic figures.

Returning to the full size and phase 0 , set the frequency of Y to 300 .


Figure 5A 2nd harmonic


5B Second Harmonic phase 0.16

Figure 5 is the second harmonic ${ }^{1}$ form. This has two loops, and as you change phase, you will see the figure "rotate" giving U shapes at 0.16 and 0.66 . If you change the $Y$ frequency to 300.07 , the figure will rotate at a slow rate. Change Y frequency to 300.8 , and it will rotate once a second. If you listen to these signals, the rotation rate will be the same as the beat rate between these slightly out of tune tones. A mistuning is equivalent to constantly changing phase. If you experiment with various frequencies for Y , you will find stable images every time Y is a harmonic of X .


Figure 6. 3rd harmonic


6C 5th Harmonic


6B 4th harmonic


6D 6th harmonic

[^0]If you set frequency Y back to 172.4 and set frequency X to the harmonics, you get the same shapes turned on their sides as in figure 7.


7A


7B

## Amplitude modulation

Adding amplitude modulation to the Lissajous generator opens up a new range of possibilities. Figure 8 shows the modification:


Fugure 8.
The tri~ object produces a triangle waveform that is used to modulate the amplitude of X Y or both, The modcontrol subpatcher (figure 9) simply allows us to switch modulation in or out leaving the amplitude at 1 .


## Figure 9.

The result of simple modulation when all three oscillators are the same frequency is not very encouraging (figure 10). The ellipse (Xphase $=0.5$ ) is simply reduced in size and pointed a bit. However, changing frequency of modulation relative to the cycles $\sim$ produces an unexpected effect. If the modulator is set to 15.1 , the ellipse will rotate like a hand of a clock.


Figure 10.
In figure 11, the cycles are both doubled in frequency so they are twice the modulator. This gives an interesting interlinked heart effect. Changing phase creates a bit of perspective.


Figure 11A


Figure 11B

Increasing the ratio to 4:1 gives doubled hearts as shown in 12 .


Figure 12A


Figure 12B

Again, changing phase creates some depth effect.
Ratios of 1:2:3 produce even more complex curves


Figure 13
If you slightly detune the Y oscillator, you will see a complex evolution from one form to another.

Looking at higher orders, you can see suggestions of the forms of the harmonic ratios of X and Y we saw before stretched into new shapes by the modulator.


Figure 14A 1:2:4


Figure 14C 1:4:5


Figure 14B 1:3:4


Figure 14D 1:4:6

You really have to see these in motion to appreciate their complexity. Here are a pair of "multiple exposures" to show how they evolve:


Figure 15A 1:2:3


15B 1:3:4

All of these figure were done with equal modulation of the X and Y oscillator. Unevenly applied modulation gives variations on the theme:


Figure 16A 1:2:2 $\bmod \mathrm{X}$


16C 1:2:4 Mod X


16B 1:2:2 $\bmod Y$


16D 1:3:7 $\operatorname{Mod} X Y=1+1.5 * t$

In example 16D the modulated Y is added to the unmodulated Y signal for smother control of asymmetry.

If we change the shape of the modulating waveform to a sawtooh (phasor~) we get a new family of curves, but have a problematic artifact to deal with.


Figure 17 1:3:3 Sawtooth modulation
The "flyback" of the sawtooth leaves an awkward straight line. (When this is done on an oscilloscope, the straight part is very thin and practically disappears.) To eliminate this we can modify the display patch to record the modulator in a 3rd channel of the jit.catch $\sim$, then detect the 0 point to blank the drawing by switching to the moveto command.

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Figure 18 1:3:3
We will still see an occasional flicker of the line, but it's not objectionable. The curves from sawtooth modulation tend to be spirals:


Figure 19A 1:1:1


Figure 19B 1:2:2


Figure 19C 1:2:3


Figure 19E 1:3:4


Figure 19D 1:2:4


Figure 19F 1:3:5

Again, slight detuning of Y gives a stately rotation. An even more amazing effect happens if you slightly detune the modulator-if its just a bit low relative to the X and Y , the spirals begin rotating, producing a sense of continual expansion or diminution.

One last trick: if small amount the modulating sawtooth is added to the X and Y signals (in addition to modulating them) There's a consistent deformation to the figure producing a definite 3-D effect. These last shots show some possibilities.


Fig. 20A 1:10 :10 with saw added to X


20B 1:3:5 with saw added to Y


20C 1:3:3 with saw on both


20D 1:3.5:6 with saw added to Y

The display patch had to be modified slightly to be able to reposition and scale the image, but these are relatively simple once the basic principles are understood.


[^0]:    1 "Harmonics" are a reference to the harmonic series, which is a simple multiplication of the starting point (the fundamental) by 23456 etc. The second harmonic of a tone is twice the frequency and sounds an octave higher. The third harmonic is three times the frequency of the fundamental and sounds a fifth above the second harmonic.

