

## Max and Filters

Filters create the "sound" of any digital composition system. After all, everything else is designed for perfect fidelity.

### Filter terms

Here are some definitions:

A filter is a circuit or software routine that changes the frequency response of a signal. The terms you really need to know are:

- **Frequency response** - this is a graph of the output amplitude produced by applying a sweep of frequencies to the filter.
- **Phase response**- this is a graph of how the phase of signals is changed at various frequencies.
- **Passband**-- the range of frequency that is relatively unaffected by the filter. If the passband is below the cutoff frequency, it is a **lowpass** filter. If the passband is above the cutoff frequency, it's **highpass**. With two cutoff frequencies, you can have **bandpass** or **notch**.
- **Cutoff frequency** -- the frequency at which the output is reduced 3 dB below the level of the passband.
- **Turnover frequency**. The frequency at which the output level begins to fall off.
- **Bandwidth**- a band pass filter has two cutoff frequencies. The bandwidth is the distance between them. Usually specified in octaves.
- **Slope** -- (or rolloff) the rate at which output declines outside of the passband. Generally quoted in dB per octave.
- **Q**- for bandpass filters, the ratio of frequency to bandwidth. High Q implies a narrow peak. Q is also loosely used in other types of filter that add a bump in response at the cutoff frequency.
- **Gain** - the change in peak amplitude of signal in the passband. Many filters increase the signal level substantially.
- **Cascade** -- a method for obtaining complex filter shapes (or steeper cutoffs) by sending the signal through a series of filters.
- **Parallel** -- applying the same signal to several filters and adding their outputs.

A digital filter works by combining a delayed signal with the original. A rough example can be created with `tapin~` and `tapout~`. If the original signal is mixed with the delayed signal, certain frequencies will be reinforced because the time it takes to get through the delay is exactly equal to the period of the signal. This will also happen at harmonics of the magic frequency. If the delay is a half period, 1.5, 2.5 3.5 periods and so on, the original and delayed signals will be out of phase and null out.

Here is the frequency response when the tap delay is 5 milliseconds:

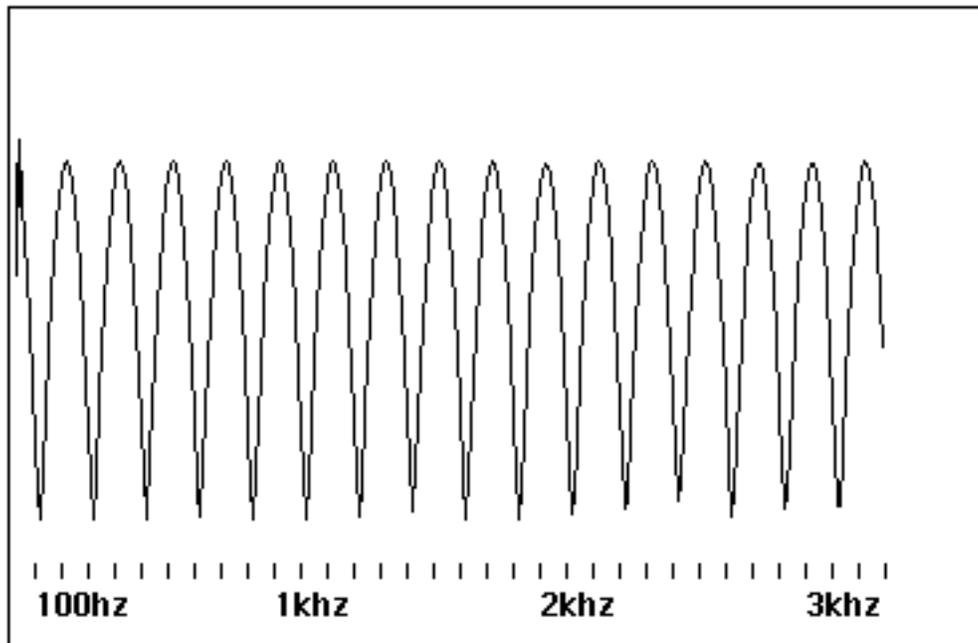


Fig 1. tapin~ to tapout~ 5ms

The maximum amplitude occurs at a frequency of 200 hz and its harmonics. The first null is 100 hz, and the others fall at the midpoints between the peaks. Note that the transition from the peaks to the nulls is gradual. This is an important feature of digital filtering, because it means that combinations of delays can be mixed to produce smooth and complex curves.

A word about the method of displaying frequency responses. These were all made with the LCD object using `average~` to measure the amplitude at each frequency. The tap delay frequency response is shown with a linear frequency scale to reveal the equal spacing of the peaks. The vertical range is from 0 to 1.0, matching the full scale amplitude in MSP. The `average~` object is set to measure rms, so a sine wave with amplitude of 1.0 will fill about 7/10<sup>ths</sup> of the screen.

These curves are often plotted on a logarithmic frequency scale. This gives each octave equal space from left to right. Likewise, the vertical scale is often in db. The other responses will be shown this way.

## Lores~

The easiest filter to use is lores~, which emulates the kind of filter found in most analog synthesizers -- a sweepable low pass response with adjustable Q (or resonance). Here is a set of responses for the lores~:

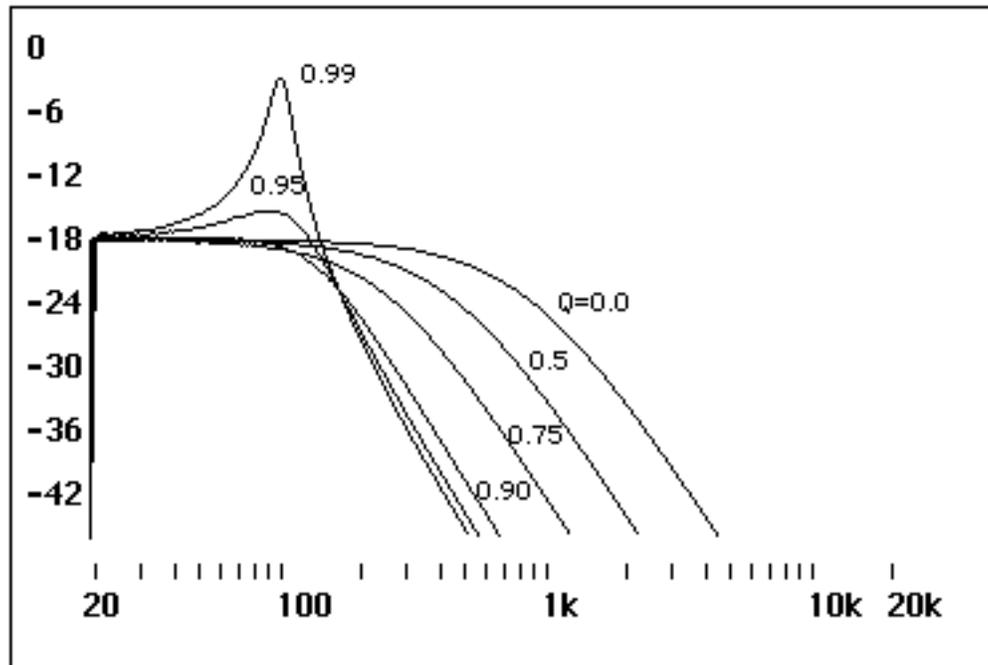


Fig 2. lores~ filter, 100hz, various resonance settings (Q)

The frequency parameter was set at 100 hz, and the response was taken for six different Q settings. Notice that the cutoff frequency ( the -3dB point) varies with the Q. This is typical of simple filter designs, analog and digital. The frequency specified would be more properly called the turnover frequency, the point at which the level begins to drop.

In filter design, Q is a number that varies from some fraction to 2000 or more, depending on the type of filter. The limit of Q will either be the point at which the circuit resonates, or where the softest practical input will distort. For lores~, Q is specified as a resonance factor, with 1.0 being the limit of Q. If you go beyond this, the filter will "explode", perhaps giving a loud distorted sound, but more likely just shutting off. To fix an abused filter, turn audio off and adjust the parameters.

Figure 3 is a plot run at 1000 hz, which follows the textbooks more closely.

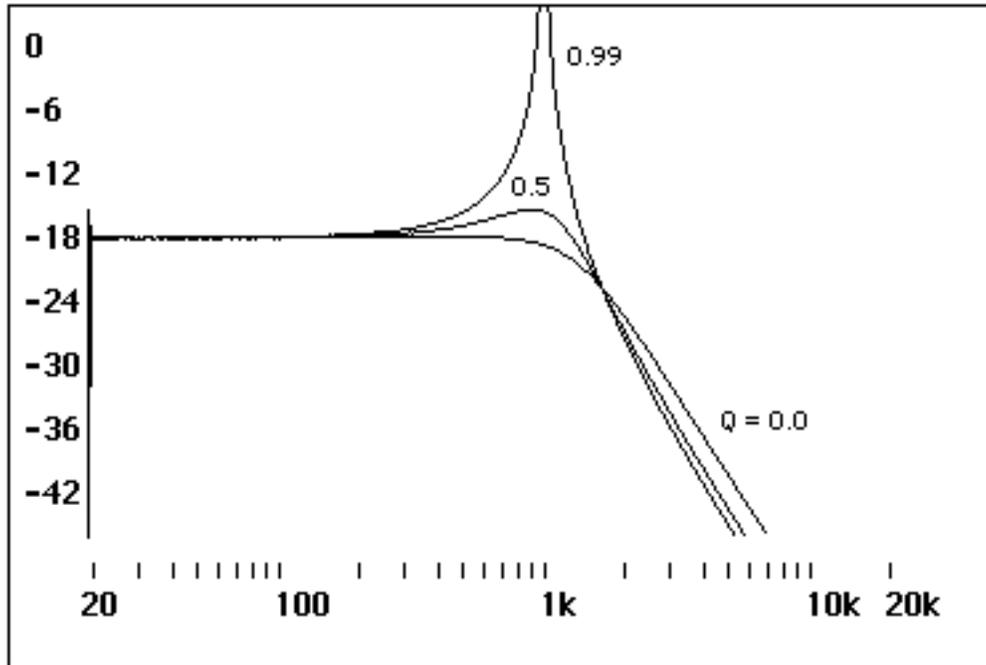


Fig 3. lores~ filter, 1khz, various resonance settings (Q)

You can also see that the height of the resonant peak varies with the frequency setting. This limits the kind of signal you can put through lores~. Typically you'll need to stay with waveforms that don't have strong high frequency partials. Here's a run with various frequency settings and Q held constant:

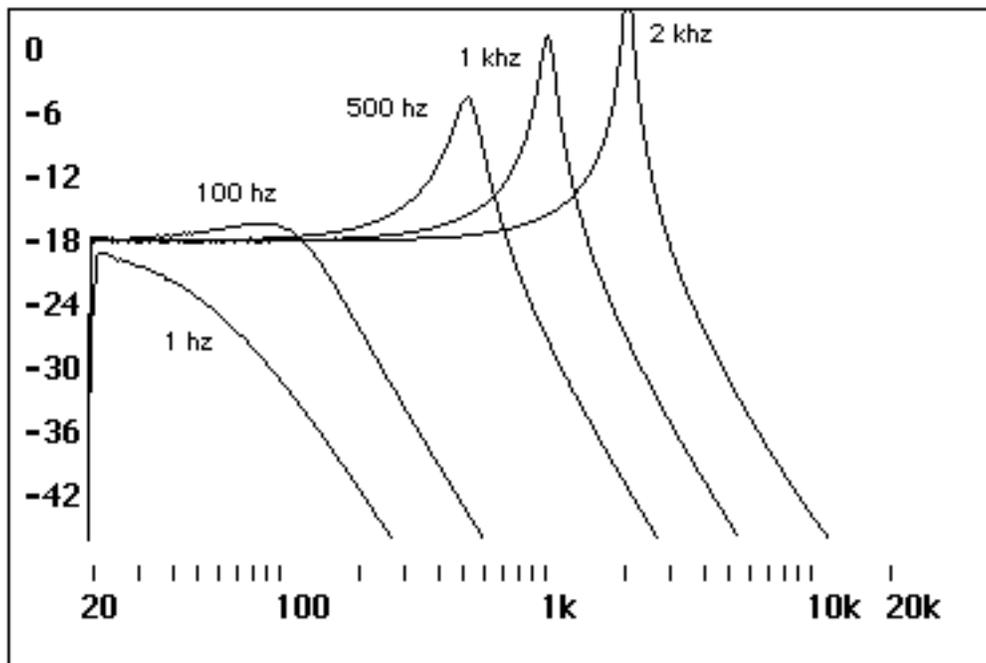


Fig. 4 lores~ filter, resonance = 0.94, various frequency settings.

As a design exercise, `lores~` would not win any awards, but it's a very interesting filter. It's very much like those on analog synthesizers. My favorite way to use `lores~` is for gating:

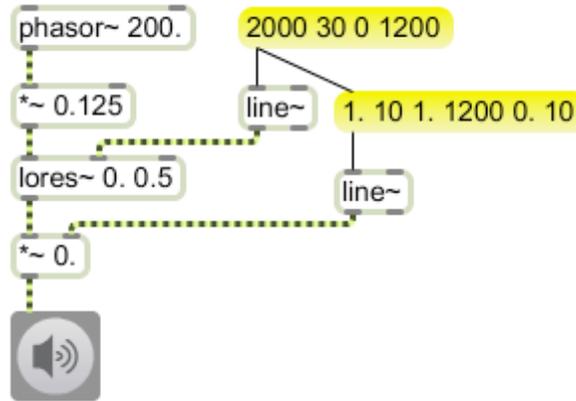


Fig. 5 Lores~ as a gate

In Figure 5, `line~` is used to produce an envelope that sweeps the frequency of the filter up and down in the classic analog synthesizer way. This gives a brassy sound. The second line is there to apply an amplitude envelope, since with the resonance factor at 0.5 a 200hz tone will still be audible when the filter frequency is 0. With higher resonance settings, the partials of the waveform are emphasized as the filter sweeps.

You always need to control levels going into `lores~`, because it very often gives back more than it got.

You can get a really funky high pass effect by subtracting the original signal from the output of `lores~`.

## Reson~

Reson~ is a better behaved version of lores~. It includes an independent control of gain to keep the resonance peak under control. Figure 6 shows the effect of changing gain.

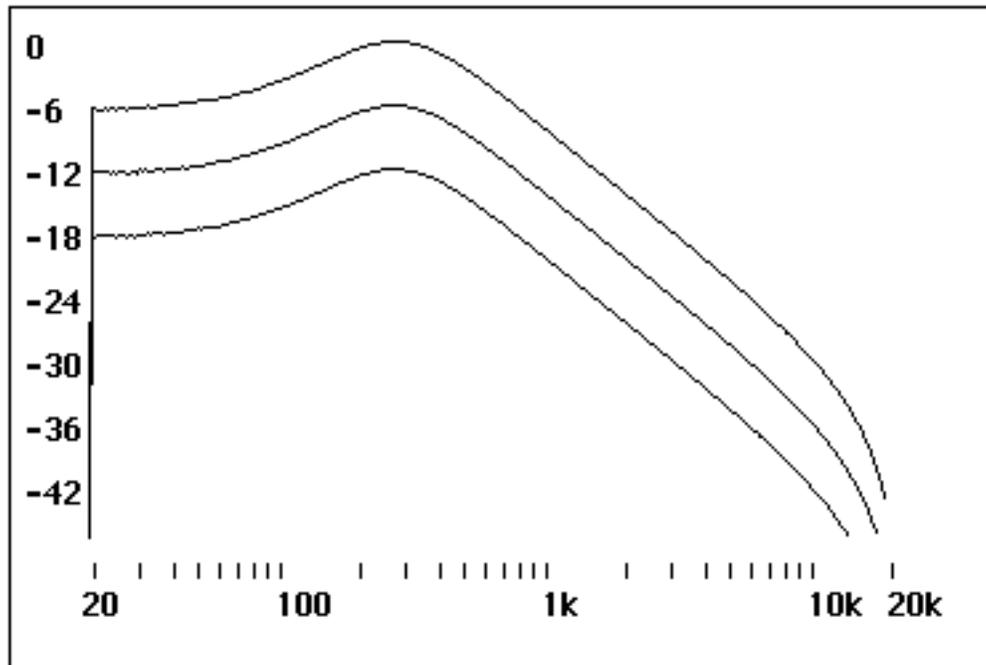


Fig 6. Reson~ gain of 1, 0.5 and 0.25

Note that cutting the gain in half produces an overall reduction of 6 dB.

Q is specified as a ratio up to 100. Figure 7 shows the effect of various Q settings. Gain is automatically recalculated for each Q setting to make the peaks equal.

The rolloff of reson~ is more gradual than lores~. This makes it a bit less effective for gating signals, unless you cascade two of them., and then the peaking effect is fairly prominent. What it really does nicely is isolate a narrow band of frequencies when the Q is high. The fffb~ object (fast fixed filter bank) is a group of reson~ type filters combined for computational efficiency. This really shines for graphic EQ type situations. The filters share an input but can be otherwise used independently.

The frequency specification for fffb~ is a little awkward, as it consists of a base frequency and then a ratio to derive the others. Here are some handy ratios:

- 2.0 for one per octave
- 1.4142 for half octave spacing
- 1.25992 for third octave spacing
- 1.2246 for 1/6<sup>th</sup> octave

- 1.0905 for 1/8<sup>th</sup> octave
- 1.059463 for equal temperament (you can play tunes!)

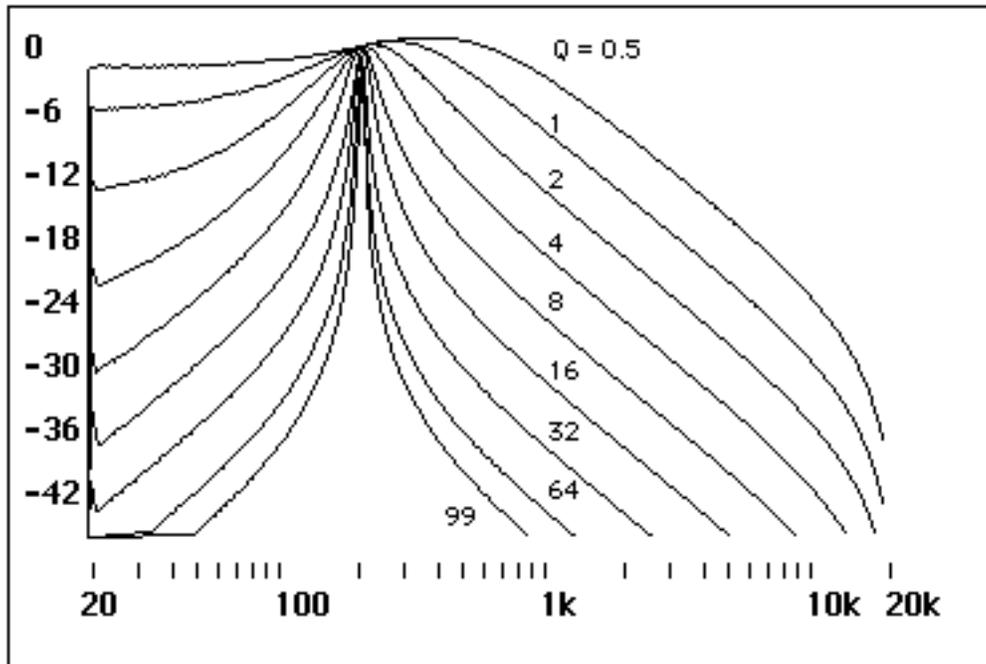


Fig 7 reson~ at 200hz and gain of 1.0 with various Q settings.

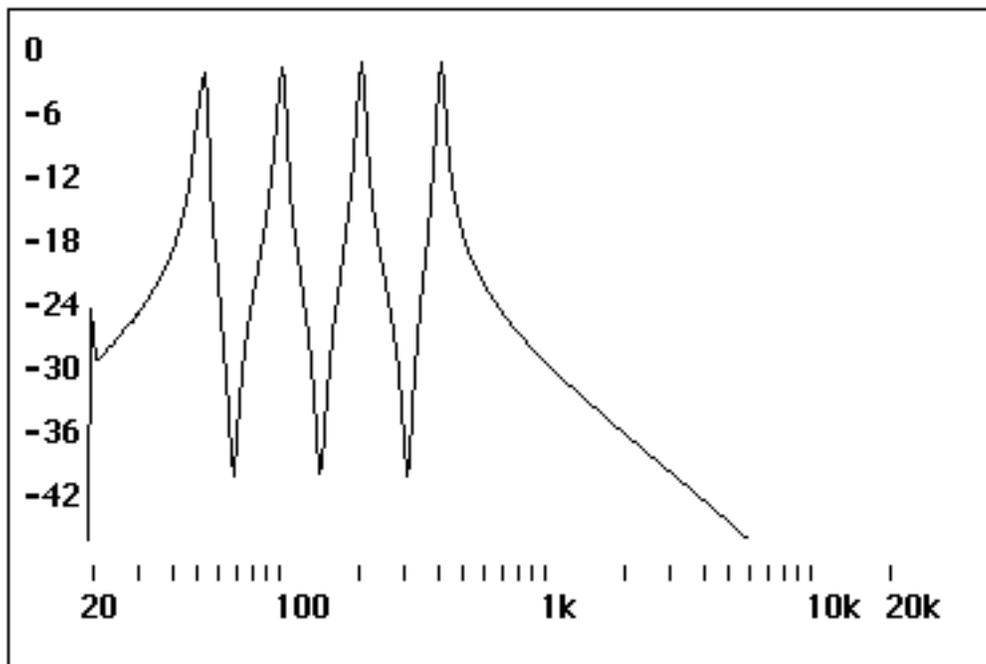


Fig 8. Fffb~ configured for 4 filters with a Q of 50 (summed)

### Svf~

The svf~ filter probably should win some kind of award. Its performance is right out of the textbooks. Svf stands for State Variable Filter, and is another circuit common in old analog synthesizers. It was usually called a multi mode filter, because lowpass, highpass, bandpass, and notch responses are available simultaneously from separate jacks. They would typically be stable at enormous Q values- the best would ring like a bell with a pulse input, but would never distort. Figure 9 shows how the four filter outputs relate to each other:

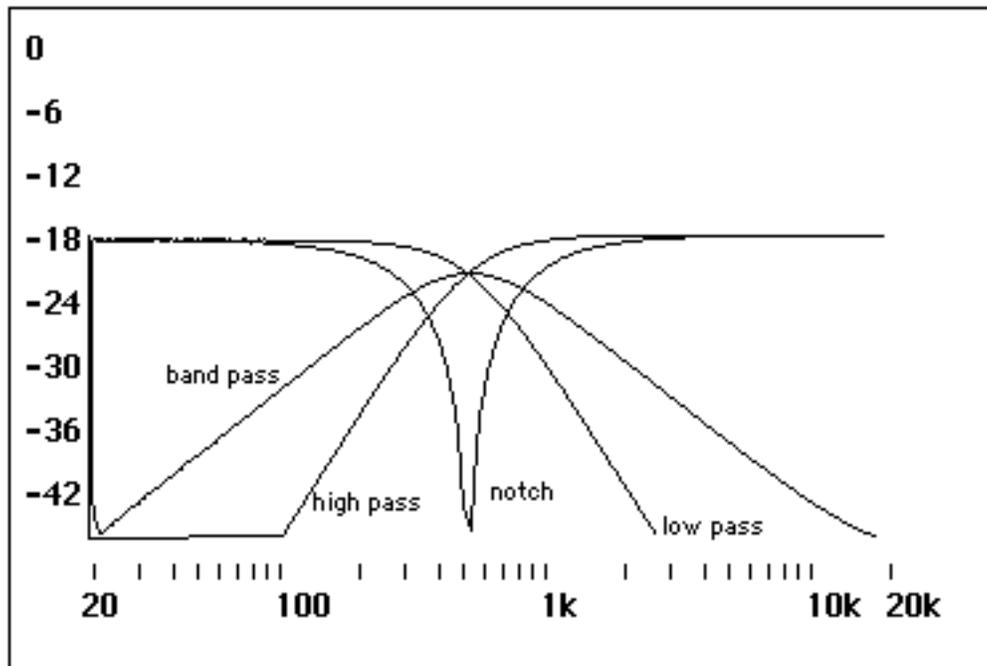


Fig 9. State Variable Filter outputs

The frequency parameter specifies the center of the bandpass, which corresponds to the turnover of the high and low pass sections and the notch. The resonance setting was zero for this plot - figure 10 shows it at 0.75:

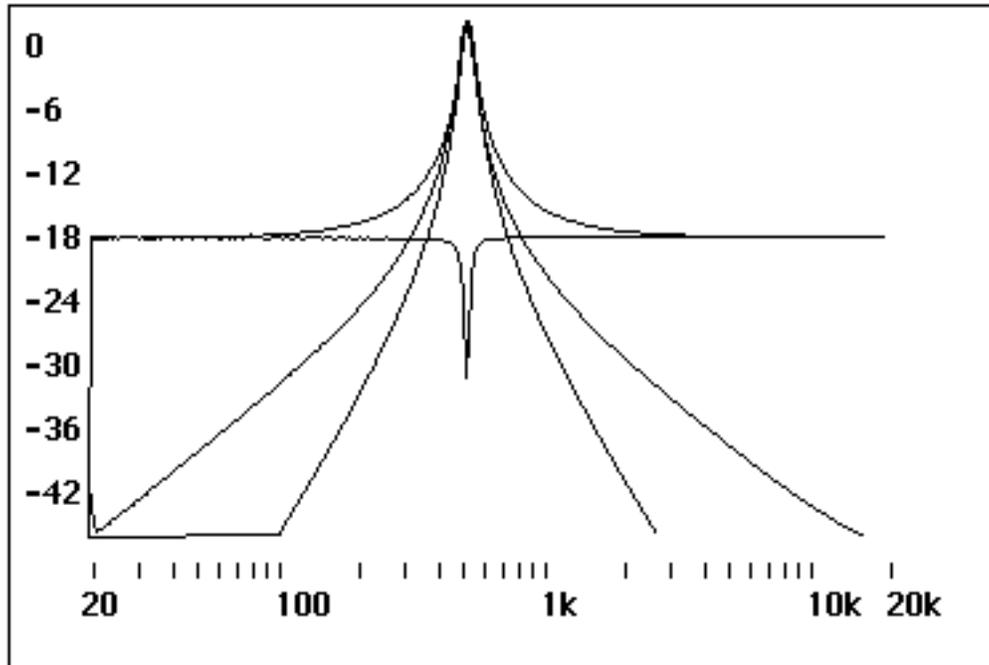
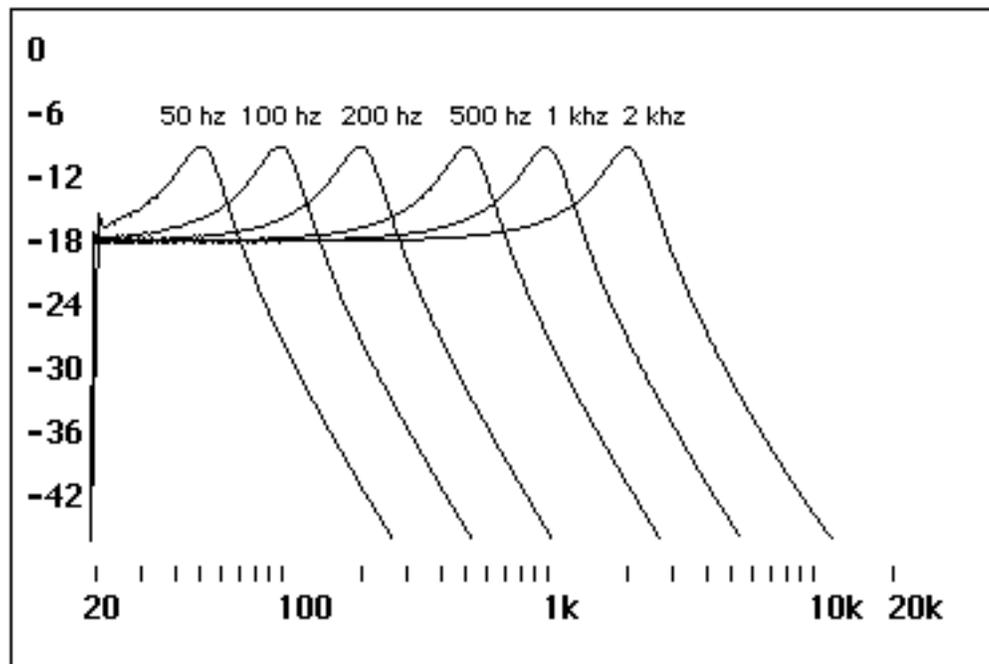


Fig 10. Resonant State Variable filter

These are nice peaks, and there is no interaction between frequency and  $Q$ . A comparison of various frequencies bears this out:

Fig 11. State Variable Filter with constant  $Q$ 

The  $Q$  bump is the same width for any frequency on the logarithmic scale.

The State variable is useful for just about any general filtering job. You can cascade two for a steep rolloff and they won't even register on the CPU meter. It is quite effective for low pass and high pass gating, although you still need to add an amplitude envelope to shut the sound off entirely.

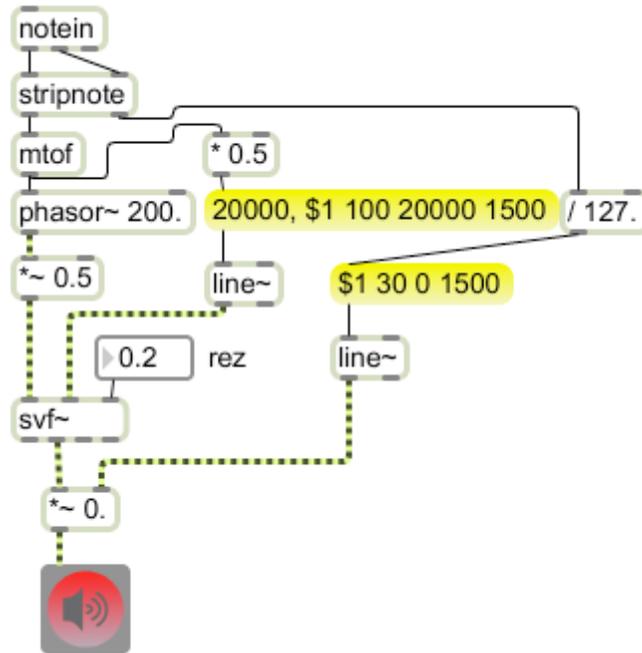


Fig 12. High pass gating with SVF

One interesting feature of state variable filters is that with high Q, they will ring on their own when pulsed. Figure 13 illustrates this. When a noteon comes in, there is a vibraphone like sound.

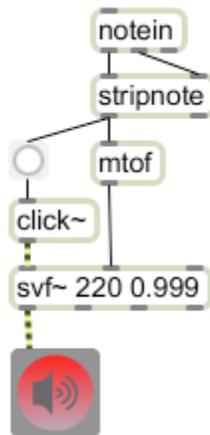


Fig 13. SVF as a tone generator.

### Biquad~ and Filtergraph~

Biquad is a do it yourself filter for those who want to really get into the numbers. Luckily for the mathematically less agile, MSP 2 features a fine design tool for biquad. This is filtergraph~.

The biquad filter is a building block that enables you to construct custom frequency responses. It will be easier to understand if we look at how the filter actually works.

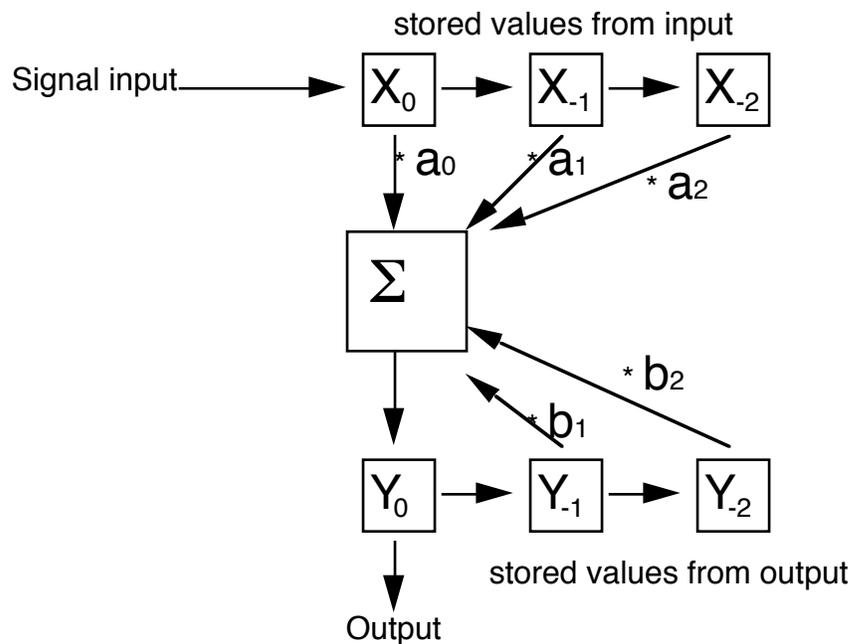


Fig 14. flowchart of biquad filter

In figure 14, the signal arrives as a series of samples. These samples are used immediately and stored to be used again later. The boxes  $X_0$  represents the newly arrived sample and  $X_{-1}$  and  $X_{-2}$  represent the stored samples. Likewise, the filter process produces a series of samples, which are passed along to the next module and stored to be used again.  $Y_0$ ,  $Y_{-1}$ , and  $Y_{-2}$  represent these output samples. You can think of the samples as moving horizontally from box to box. The process is really very simple: each of the samples mentioned is multiplied by some coefficient and they are all added together to produce  $Y_0$ . The coefficients used on the input samples are called  $a_{\text{something}}$  and the coefficients for the output samples are  $b_{\text{something}}$ .

In digital filterspeak the equation for this is

$$Y[n] = a_0 * x[n] + a_1 * x[n-1] + a_2 * x[n-2] + b_1 * y[n-1] + b_2 * [n-2]$$

The terms derived from storing input are called "zeros" and the terms derived from storing output are called "poles". Most digital filters can be described this way. For instance the tapin- tapout pair just has a zero at  $x[n-220]$  (It should be 220.5 for 5 ms at 44.1 khz sampling rate, but you can't have delays of part of a sample.)

Biquad implements a two pole, two zero filter. The filter type, frequency, gain and Q are all determined by the coefficients. Unfortunately, the relationship between, say,  $a_2$  and frequency is not simple and straightforward. All of the coefficients interact in a very complex way<sup>1</sup> to set the filter properties. It takes a computer program to work it all out. Filtergraph~ contains the program.

Looking at the filtergraph~ help file will give you an idea of what it can do. Start with the display on the right of the screen -- choose a filter shape, then play with the frequency and gain boxes. You will see how the shape can be changed. You can also change frequency and gain with the mouse. Click on the curve near the turnover point, and the point will follow the mouse. The corresponding coefficients are shown at the bottom of the window. Note that all change as the frequency and gain are moved. These can be dumped directly into biquad~ so you can listen to the effects of the changes. In fact, the help window does just that.

The filtergraph~ inspector has a lot of important settings, starting with the type of filter you are designing. Other highlights:

- Gain enabling allows the mouse to change gain when moved up and down. If this is not checked, gain will remain at 1.0.
- The constraints limit the range of the frequency and gain parameters.
- The frequency display sets the range of frequency that will be shown, as well as choosing log or linear display.
- A full spectrum shows how the frequency response is wrapped into the negative side of the spectrum. It's an interesting theoretical trick.
- The amplitude display lets you set the range of amplitude values that will fit on the display. You can also choose linear or log (dB) response

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<sup>1</sup> It's like this:

With frequency F and resonance Q

First find the angular frequency with  $\omega = 2\pi F/SR$

Find  $\alpha = \sin(\omega)/2*Q$

Then for a lowpass filter

$A0 = (1 - \cos(\omega))/2 / (1 + \alpha)$

$A1 = 1 - \cos(\omega) / (1 + \alpha)$

$A2 = A0$

$B1 = - 2 * \cos(\omega) / (1 + \alpha)$

$B2 = (1 - \alpha) / (1 + \alpha)$

(thanks to R Dudas for the math)

- Phase response can be as interesting and important to know as amplitude response. Take a look at the phase response of any notch setting.
- Output coefficients on load lets you save filter settings with the patch.
- Colors are always fun to play with.

The graph on the left of the help window will follow along with your changes, as it is in display mode, converting coefficients to a response curve. If you get coefficients from somewhere else, such as a book on digital filter design, display mode will show you what to expect.

Display mode can also show you what will happen when biquad filters are cascaded. Double click the [p cascade] box in the help window to get an example. This represents an EQ made of a low shelf, four peak/notch and a high shelf filter connected in series. To hear this, you would send the coefficient list from each of the small filtergraphs to a set of biquads wired left to right. The cascade command to a filtergraph in display mode will make it combine the responses graphically to reflect the effect of a series connection.

If you cascade two or more biquads with the same coefficients, you get a generally steeper cutoff slope. In theory, you should be able to adjust the frequency and gain of each filter slightly to change the corner response (that's what happens right at the cutoff frequency), but it doesn't work well and doesn't make a whole lot of audible difference.

You can use filtergraph and biquads for sweeping and gating effects. Use line (instead of line~) to apply envelopes directly to the frequency input. The graphic display may not keep up<sup>2</sup>, but it won't affect what you hear. Two cascaded biquads can give a nice Moog-like sound as shown in figure 15. (Max 5 introduced an easier way to cascade biquad filters-- see Cascade~ in the new section of this essay.)

**Filtercoeff~** provides most of the functions of filtergraph without the display. You can use it to control standard filter types with signal rate accuracy. There are quite a number of types available, so the inability to draw your own is a minor limitation.

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<sup>2</sup> In fact, you may want to hide the filtergraph~ in a subpatcher to save space.

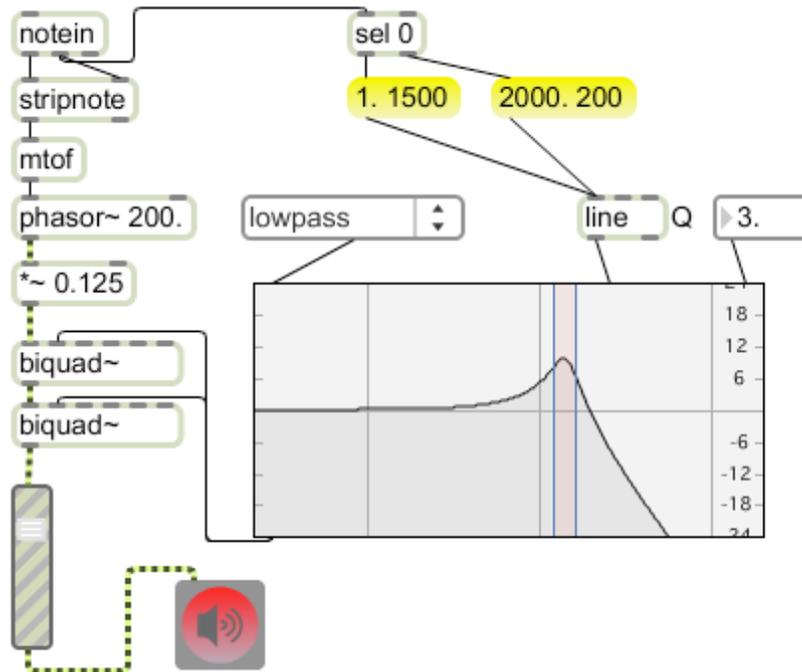


Fig 15a. Gating with biquads.

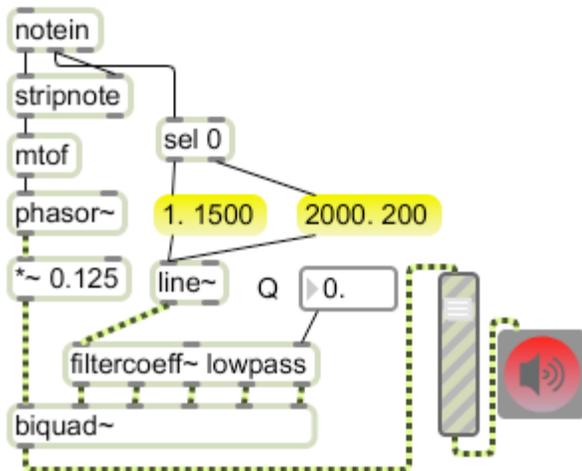


Figure 15b. The same patch with filtercoeff~

## Allpass~

Allpass is an interesting filter. It does not change the frequency response at all -- it's dead flat. What is interesting about it is the Phase Response. Most filters change the phase of the signal in some way. Generally the change is fairly subtle, at least in the passband. Allpass~ changes phase in a very dramatic fashion. Figure 16 shows the output of allpass~ added to the original input to give an indication of how the phase changes.

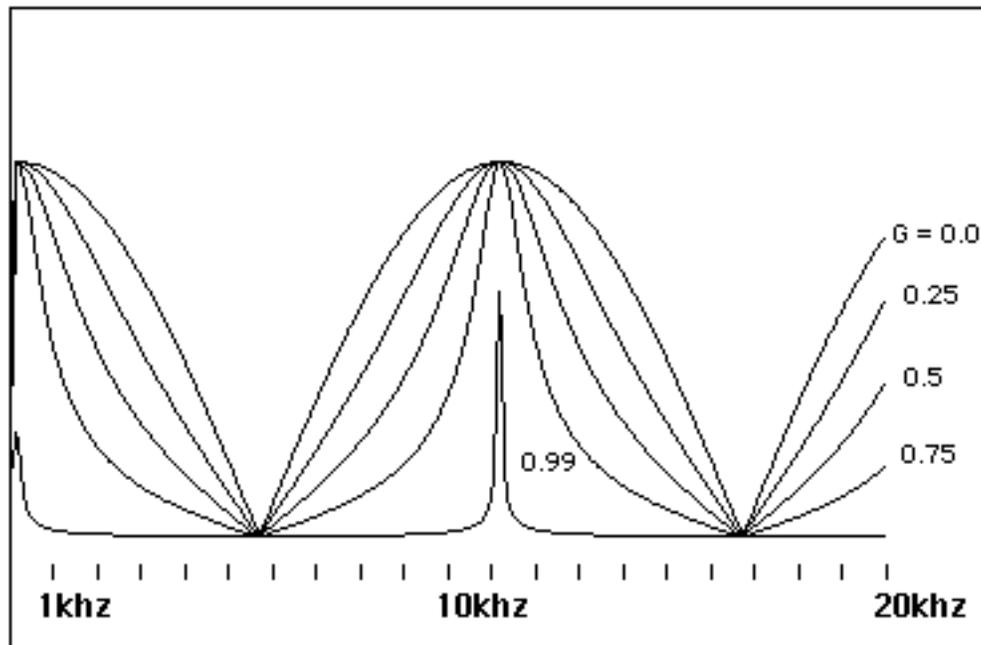


Fig. 16 AllPass~ phase response at various levels of gain

This plot was taken with a delay time of 4 samples. If you follow the top curve as the frequency increases, you will see that the phase changes from 0 to 180 degrees ( $0$  to  $\pi$  radians) at 5.5 kHz. It continues to change to  $2\pi$  at 11 kHz, is  $3\pi$  at 16.5 kHz, and would be  $4\pi$  at 22 kHz if the graph went that high. Figure 16 shows only the amplitude of processed + unprocessed signal, so the drawing is folded back into itself. This is similar to the tap delay example - in fact it's the same at a higher frequency. Increasing the gain makes the phase changes more sudden, which sharpens the response peaks in this example. We can't hear phase on steady sounds, but with transients there are some mysterious lingering tones as the help file demonstrates. Even better is to change the delay time, as figure 17 illustrates.

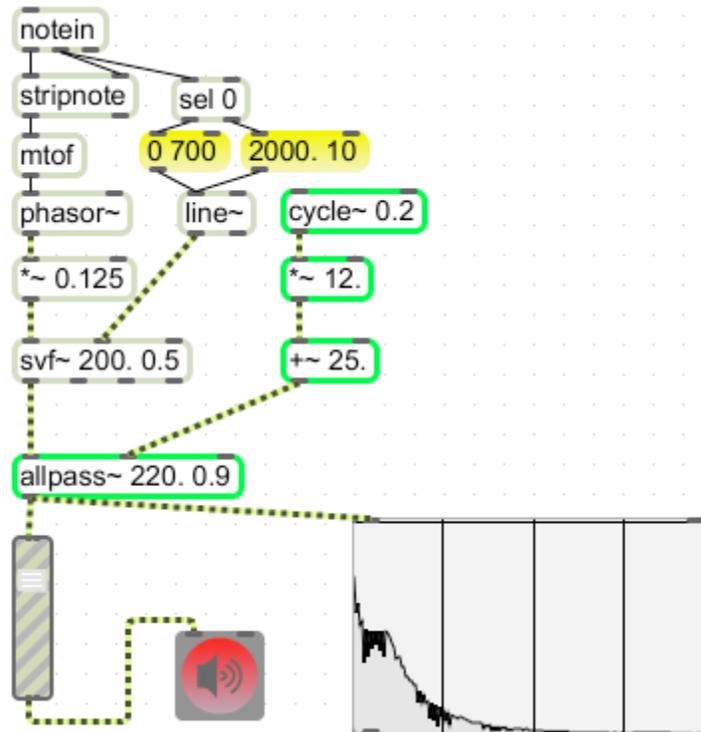


Fig 17. Flanging with allpass. The dark areas in the spectrum are notches.

### Comb~

The `comb~` filter is similar to `allpass~` but adds some features. First, feedforward (the trick of adding the original to the processed signal to get the phasing effects) is built in. Second, there is also feedback, which runs some of the processed signal through again. (as with any feedback, levels have to be watched carefully. Here the limits are -1.0 to 1.0). figure 18 shows the effects of feedforward and feedback.

With no feedforward or feedback, you get a nice flat allpass response. Adding feedforward gets the same deep notches we have seen before. Feedback alone is similar, but you just get dips instead of notches. Negative values for feedback flip this curve upside down. With both feedforward and feedback, the peaks jump through the roof.

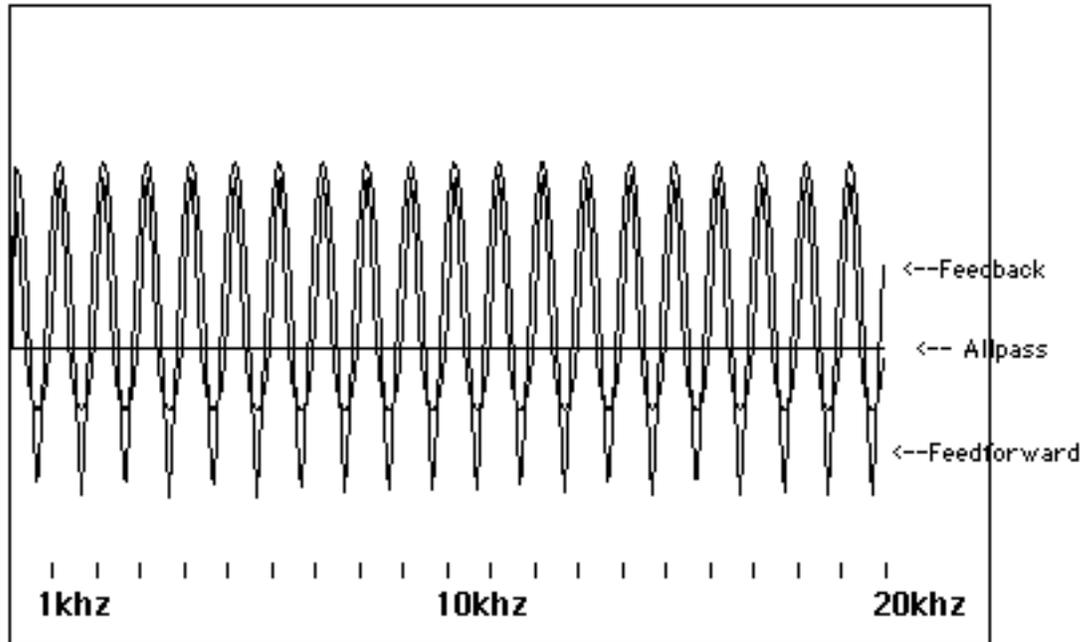


Fig 18 comb~

With the feedback up, there is a lingering effect as the echoes of any input die down. In fact, with a pulse input you can make an instrument with a very rich twangy sound.

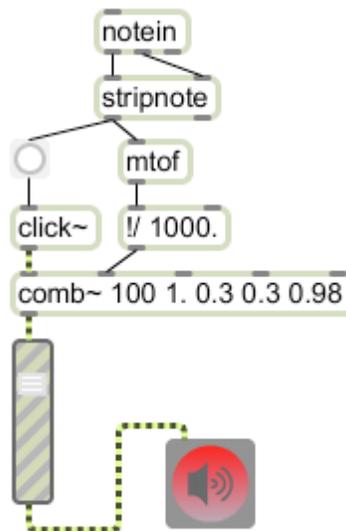


Fig 19. Comb~ as a signal generator.

In figure 19a the frequency is converted to a delay time with the `! / 1000.` object.

The `teeth~` filter is very much like `comb~`. It adds independent control of feedback and feedforward delay times. Figure 20 shows a `teeth~` response with a feedforward delay of 2 ms and feedback delay of 1 ms.

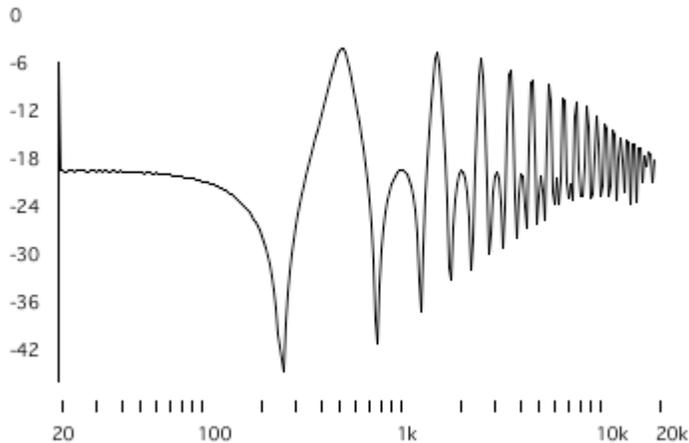


Figure 20 . Teeth~

The following pages cover filters new since the first edition of this tutorial.

### Buffir~

Buffir~ filters by convolution. Convolution would fill a book-- in fact many books (Amazon lists 2389 titles for Digital Signal Processing, and any decent DSP book must cover convolution.). Here's a short version:

To produce a convolution of signal A by signal B, multiply the entirety of signal A by the first sample of signal B and reverse the result. Now do it again with the second sample of B and add the result to the previous result. Keep going until you have used up all of B. Another way to look at it is as a series of one sample delays with the values of B as coefficients. Apply A to the delay-- when the first sample of A reaches the end of the chain, add up all of the delay outputs. This can be a continuous process, if the computer can keep up. Luckily there is a short cut-- just take the FFTs of A and B and multiply them-- the IFFT of the result is the convolution. (If you haven't seen it yet, take a look at my Fourier Notes tutorial and the discussion of Forbidden Planet.)

Convolution is applied to filtering by finding the impulse response of the filter we want to emulate. The impulse response is what you get if you send a 1 through the filter followed by a lot of zeros. This is equivalent to popping a balloon in a nice resonant church. (Impulse responses are good for modeling reverberation, too.) Of course, there are mathematical ways of finding impulse responses, but I won't go into that here<sup>3</sup>. Interestingly, you can now download impulse responses, but they are mostly appropriate for reverbs and won't work well in buffir~<sup>4</sup>.

<sup>3</sup> In fact, I have no idea.

<sup>4</sup> You can always buy a plugin like SIR2, Meatscience or Altiverb.

Convolution filtering is best suited to coloring sounds. If you use the response of a shower stall for `buffir~`, sounds run through it will take on something of a showery character. Figure 21 shows a patch for playing around with impulse responses. `Buffir~` requires a matching `buffer~` with the impulse response in it. The patch adds editing to the `buffer~` in the same manner as the wavetable synthesizer patch. You can load a response in the buffer, or draw your own. Actually, I find the most interest in slightly modifying a preloaded impulse.

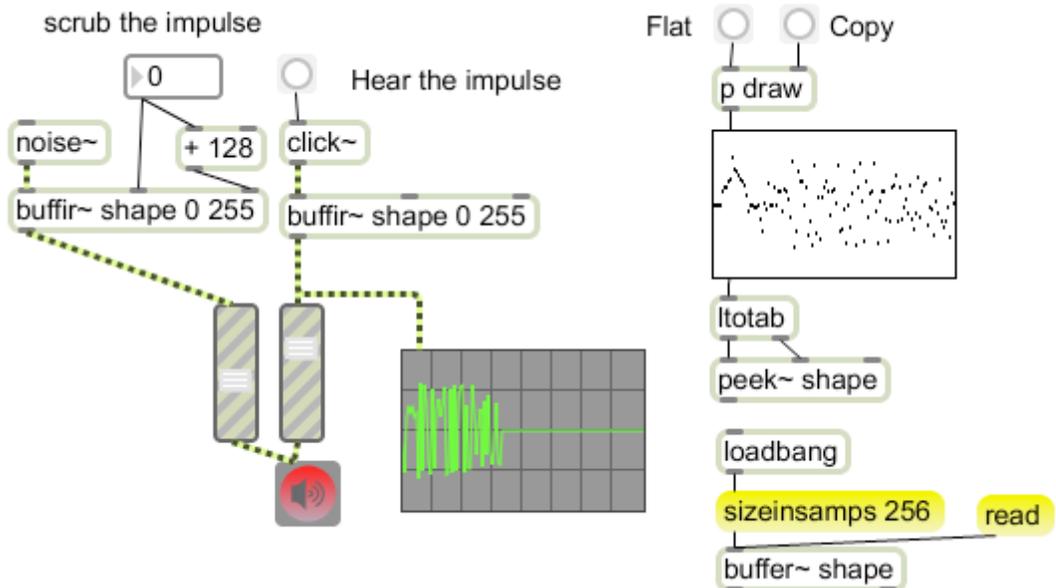


Figure 21. Playing with `buffir~`

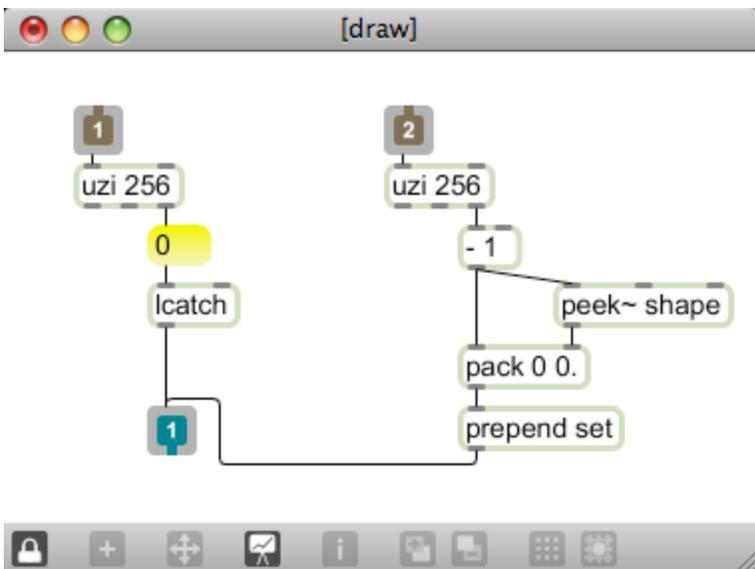


Figure 22. The draw subpatch.

### Cascade~

Figure 15 used two biquad~ filters to produce a steeper cutoff than one section alone provides. I was in fact cheating by sending the same set of coefficients to both filters. Normally, when biquads are combined, the coefficients of each are tailored to the application. Cascade~ lets us do this. By informing the fintergraph~ of the number of sections needed, you can draw extremely complex curves and have them perfectly realized. Figures 23 and 24 illustrate. We generally use filters of this type for EQ and coloration.

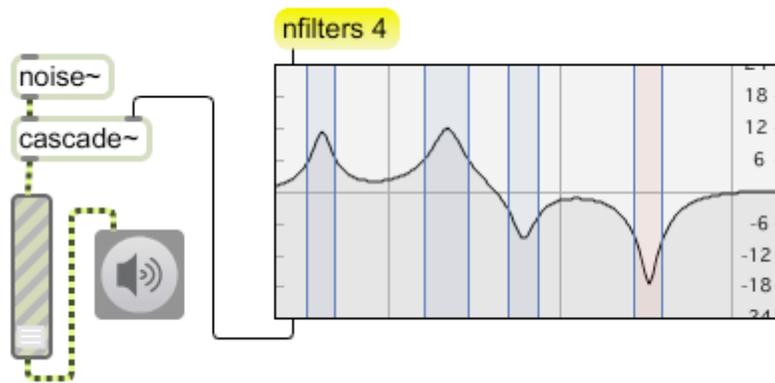


Figure 23. A four stage biquad equalizer.

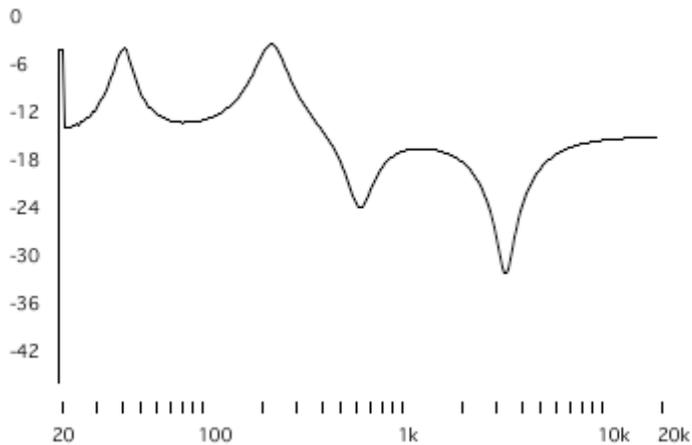


Figure 24. Response of figure 23.

### Cross~

There's not a lot to cross~, just matching lowpass and highpass outlets.

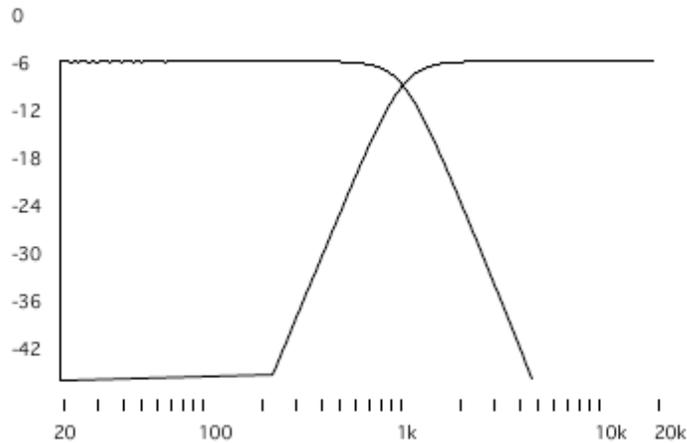


Figure 25.

Figure 25 shows the response of both outlets. The highpass response is a perfect mirror of the lowpass. The 18 dB per octave slope makes it an excellent lowpass gate when used in a patch like that of figure 5. The transition is quite smooth with no hint of a quack.

If you cascade two cross~ objects, taking lowpass from one and highpass from the other, you can get a fine bandpass like figure 26. This can be used in a poly~ voice to produce very pleasant, organ-like tones. Figure 27 shows an example. The harmonic control has a surprising amount of effect on the timbre.

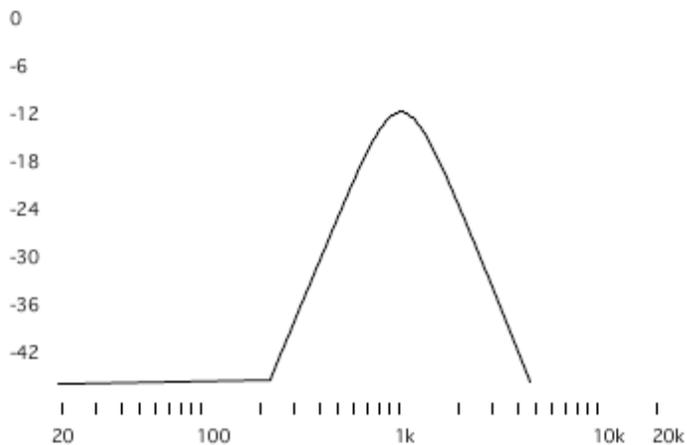


Figure 26. Two cross~ filters in series.

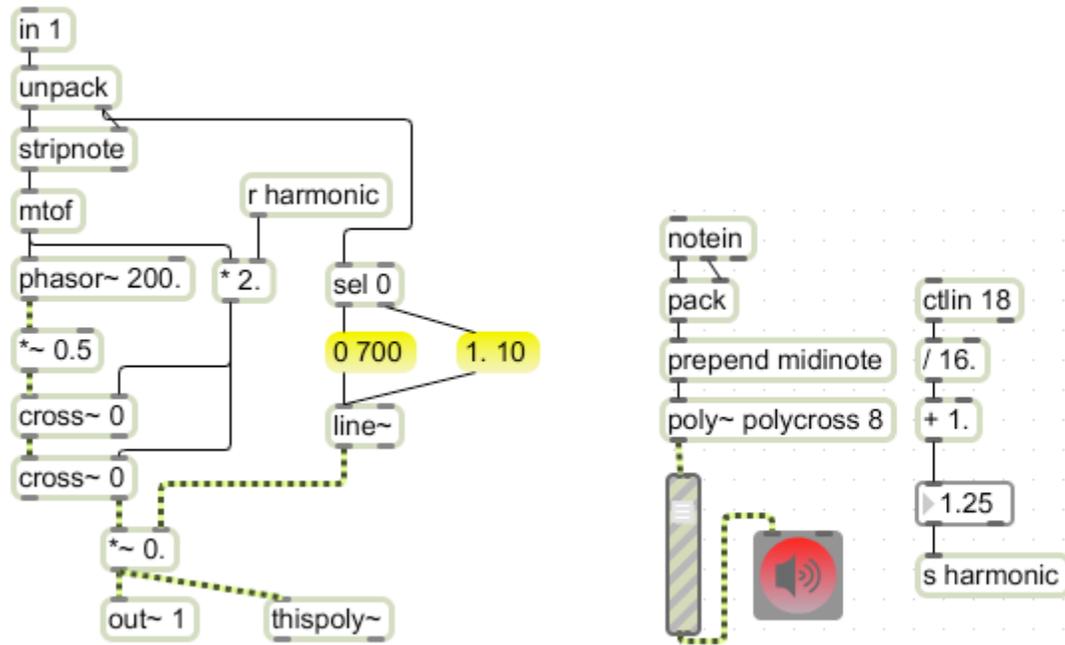


Figure 27. polycross voice and player.

### Hilbert~

The `hilbert~` object is an allpass filter that changes the phase but not the amplitude of the input. There are two outputs. The signal from the left has the phase of all components delayed by 45 degrees<sup>5</sup>. The signal from the right is the signal advanced 45 degrees. Therefore, the left and right outlets are 90° out of phase. The analog version of this is known as a quadrature filter. If you listen to either output, you will hear no change from the original-- if you combine the two, it is merely a bit louder. The use of this only becomes apparent when you apply some phase related process to the two outputs. For instance, this process is used to demodulate short wave radio.

The classic music application for this is the frequency shifter. Figure 28 shows how it works. (This is in the help file, too.)

<sup>5</sup> Note that in `allpass~` and `phaseshift~` the phase shift is dependent on frequency, since it is actually a delay. The phase shift in `hilbert~` is  $\pm 45^\circ$  at all frequencies.

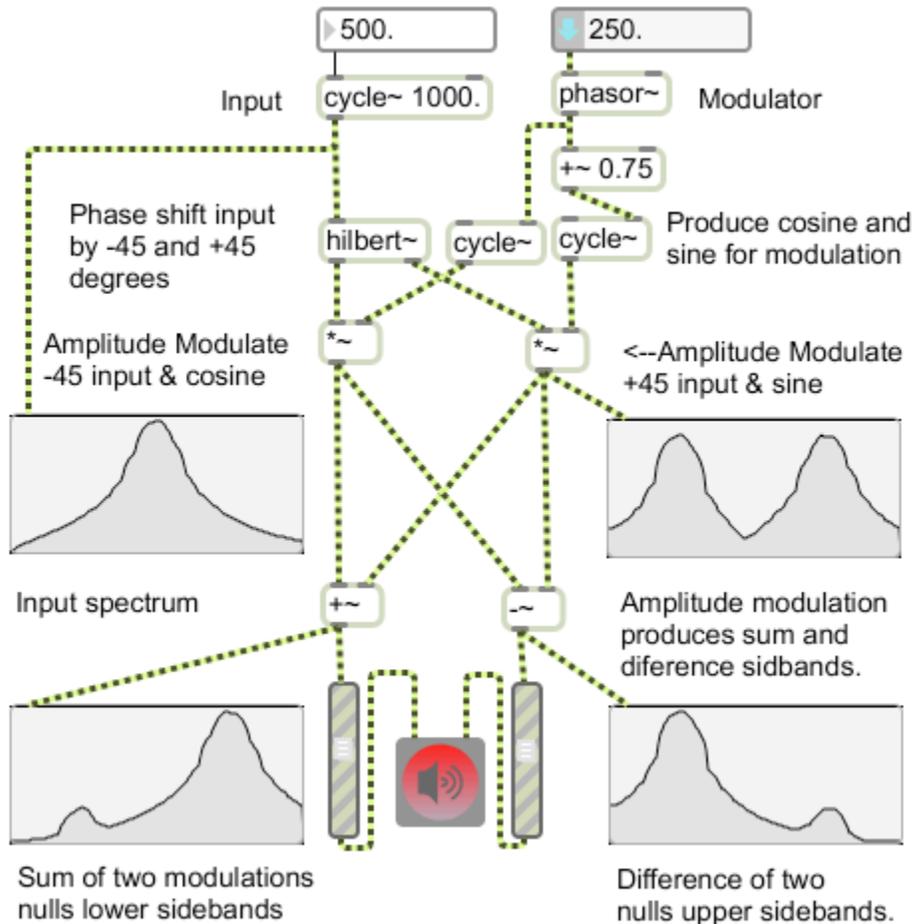


Figure 28.

If we amplitude modulate (multiply) each side of the signal by signals that are also  $90^\circ$  out of phase, we get the usual sidebands at frequencies of input + modulator and input - modulator. The input (carrier) is completely eliminated if everything is working right. The two results sound alike, but the lower sidebands on the cosine side are  $180^\circ$  out of phase with the lower sidebands of the sine side. Thus if you add the modulated signals the lower sidebands are nulled out. Subtracting the signals nulls the upper sidebands.

The result is a strange sound. All components of the input are shifted by a constant frequency, so a tone with components of 100, 200, 300... winds up with something like 120, 220, 320... In other words, harmonic sounds become clangorous. The shift up is particularly useful, producing sounds like the villains in Dr. Who.

### Onepole~

Onepole is the simplest of filters, providing a 6dB/octave lowpass response. This comes up often enough that DZ provided a dedicated low-load object<sup>6</sup>, similar to a simple capacitor-resistor pair in the analog world. The response is illustrated in figure 29.

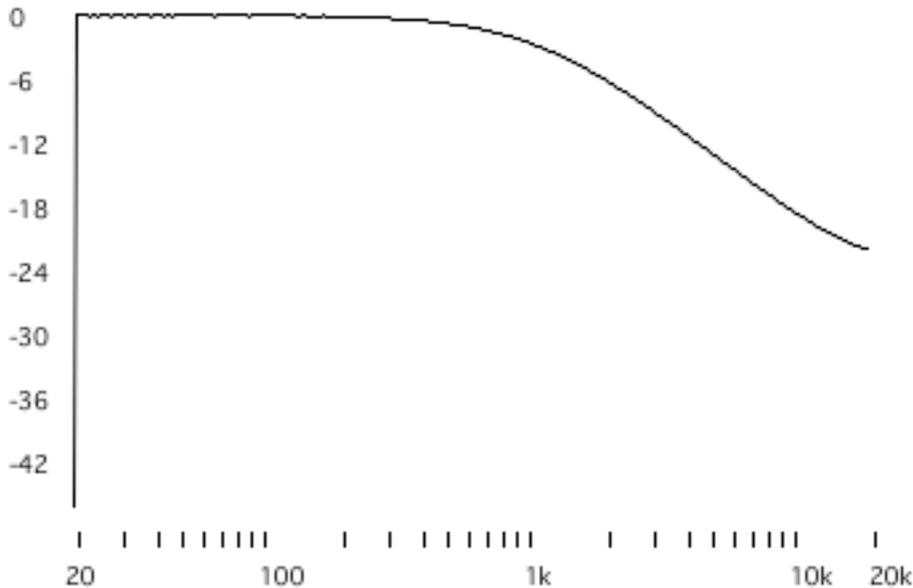
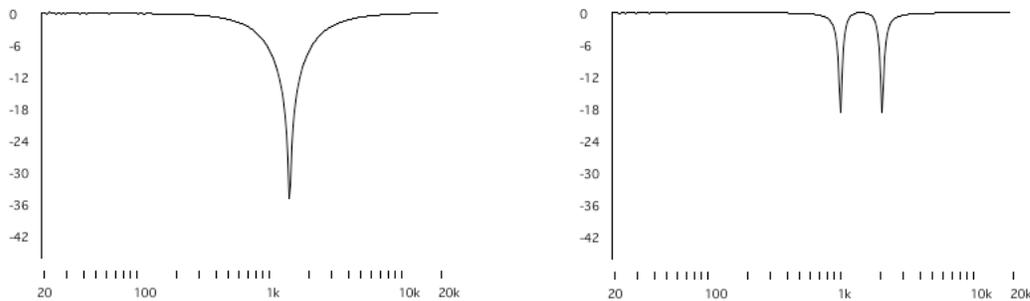


Figure 29.

### Phaseshift~

Phaseshift~ is well named. It is an allpass filter that shifts components near the turnover frequency by 180°. Combining the output of phaseshift~ with the original signal will produce a notch. High Q settings (range is 0 to 100) narrow the notch. Since the frequency is actually the point of 90° phase shift, a low Q will move the center of the notch up a bit. Two phaseshift~ objects in series will produce a double notch. Figure 30 illustrates.



$$F = 1000 \text{ Hz} \quad Q = 0.5$$

$$F = 1000, F = 2000 \text{ Hz} \quad Q = 5.0$$

Figure 30.

<sup>6</sup> Hence the old saying, never use a biquad when a single pole will do.

Most phase shifting devices provide an LFO to modulate the notch frequency. One approach is shown in figure 31.

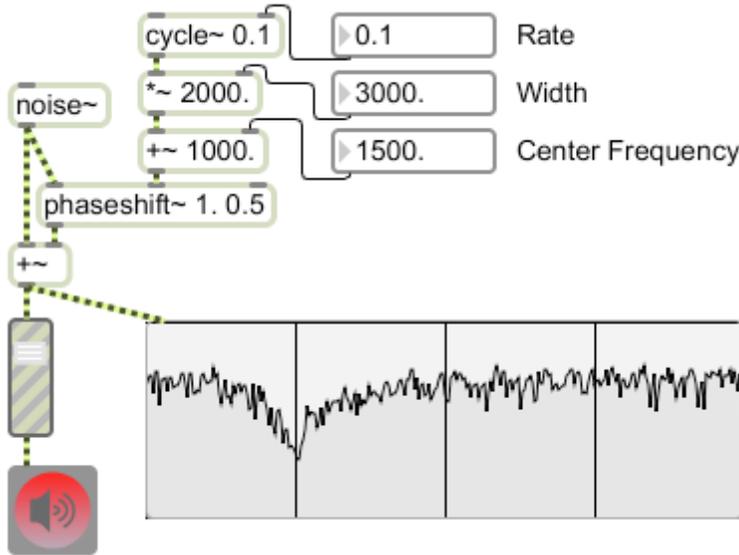


Figure 31.

**Slide~**

Slide reduces the rate of change in signal values. When the signals are audio, this is a lowpass filter, when the signals are sub-audio, it is an integrator. The classic application of integration is to extract the amplitude envelope from a sound. In Figure 32, the input audio triggers the sound of an oscillator. The arguments to slide~ are the divisors used in the change formula. The right section of figure 32 shows the relationship to the input and output. When the input changes, the output is only allowed to change by change/divisor on the first sample, then remaining-change/divisor on each following sample. This produces a log curve as the output approaches the input in ever-decreasing steps. Positive changes and negative changes use different divisors.

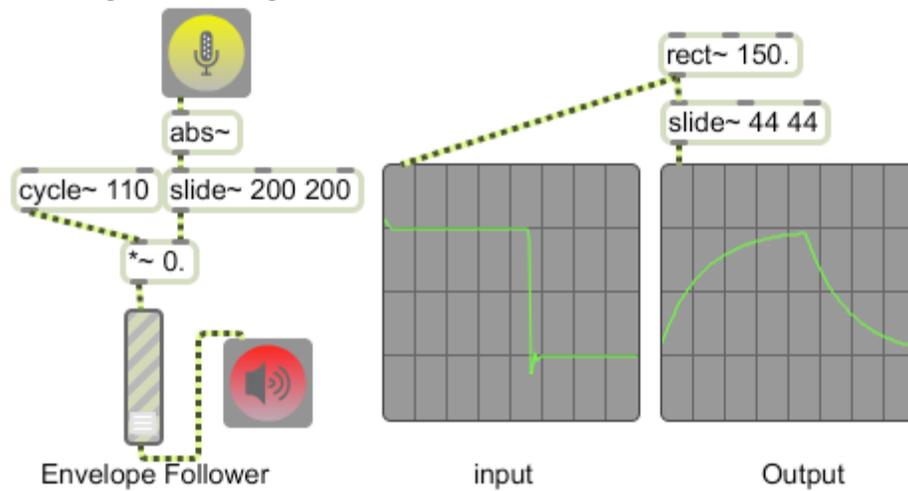


Figure 31. Slide~

**Summing up**

It's easy to get involved with the minutiae of filter specifications and math, but the important thing about any filter is how it sounds. You should try them all in all of the major applications:

- Coloring synthesized sounds
- Gating synthesized sounds
- EQ of input sounds
- As resonant objects
- In analysis, such as pitch detection or vocoding.

I have my own ideas about what works best in each of these situations, but you should come to your own conclusions. Just start with low levels and modest parameter values until you get a handle of what can be pushed and which way.