Making Waves With Frequency Modulation

The Theory

John Chowning published his first paper on FM synthesis[1]in 1973 This was a major breakthrough because it used the computer very efficiently to create a wider variety of sounds than had been available. FM synthesis was patented by Stanford University, and an exclusive license to build FM instruments was eventually sold to Yamaha. Although it formed the heart of the most popular synthesizer ever made (the DX-7) the system proved too difficult for the average musician to understand, and eventually samplers became the central instrument in most studios.

The heart of the technique is the way extra tones (sidebands) are created when one oscillator is used to modulate the frequency of another. These sidebands are symmetrically spaced about the frequency of the carrier, and the size of the spaces is equal to the frequency of the modulator. I is the <u>index of modulation</u> which is the ratio of the deviation of the carrier to the frequency of the modulator. (In other words, if you have a modulation frequency of 100 hz, and the amplitude of the modulator is enough to add 50 Hz to the carrier, the Modulation Index is 0.5.) Increasing modulation increases the number of sidebands, but the amplitude of the sidebands varies in a rather complex way as the modulation changes.

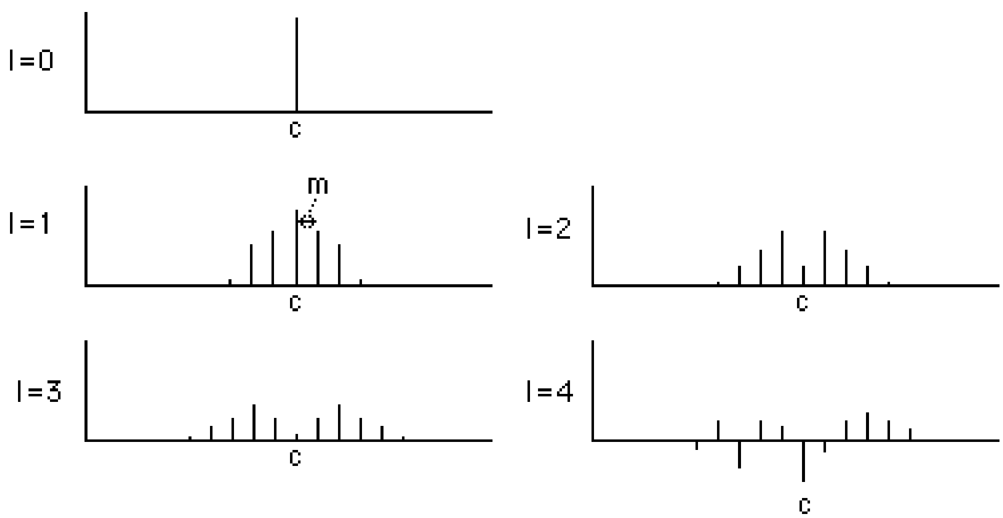


Figure 1. FM spectra in theory

In Figure 1 the carrier is 1000hz and the modulator is 100 hz. As the modulation index (I) increases, additional components appear at a space of 100hz above and below the carrier. When I=1 there are components at 700, 800, 900 , 1000, 1100, 1200 and 1300. Some of the components may be 180° out of phase. For I=4 I have shown the phase by drawing

[1]Chowning, John M. "The Synthesis of Complex Audio Spectra by Means of Frequency Modulation."
Journal of the audio engineering society, vol.21 (1973), pp. 526,534. Reprinted in
Roads, Curtis and John Strawn eds. FOUNDATIONS OF COMPUTER MUSIC. Cambridge MA, MIT press, [Available in Mc Henry]

the component below the bar. (Most drawings show the absolute value of each component.) The general shape of the amplitudes follows a family of curves known as Bessel curves.

Do it in Max

Here's a Max patch to explore FM. It's built to be triggered by the receive objects: freq to set the frequency, nON to turn the sound on and nOFF to turn the sound off. The modulation index is set with a number box. The frequency of the modulator is set as a ratio to the note frequency. This gives a consistent sound as the pitch is changed.

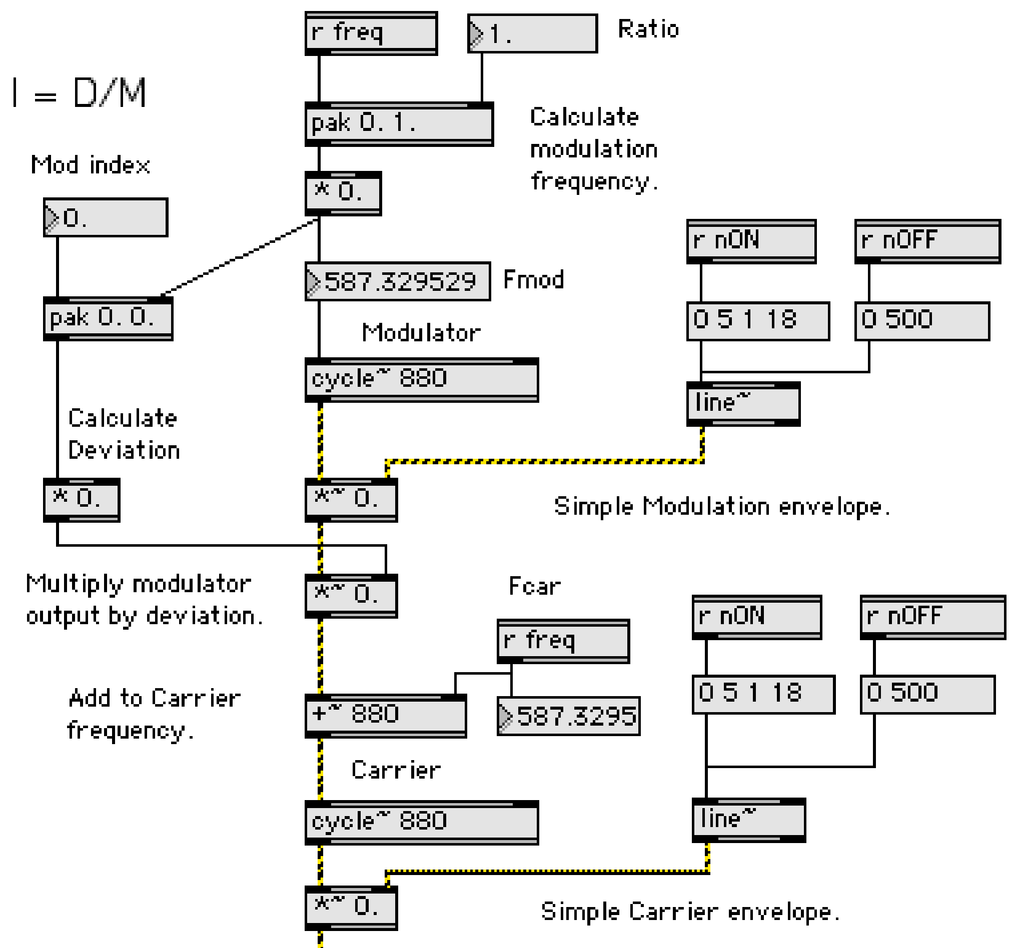
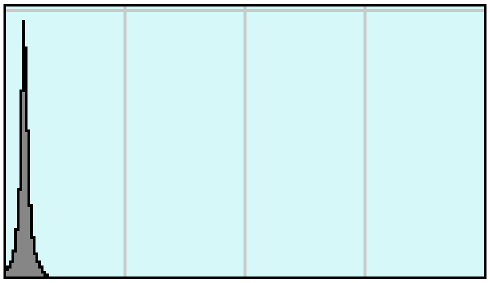
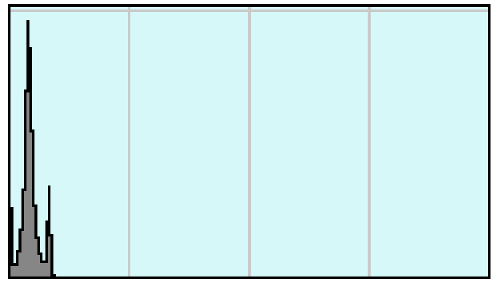


Figure 2 The basic FM patch

Figure 3 shows spectra generated by this patch. In all of the examples, the ratio was set to 1, so there's a simple harmonic relationship of the carrier and any sideband frequencies that appear. There are components at 880, 1760, 2640, 3520 and 4400 Hz, with a hint of 8800. There's also a barely visible component at 0.

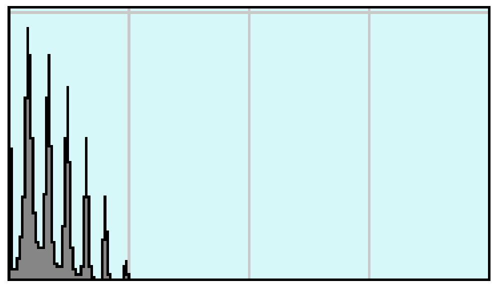


Ratio 1.00 Index 0.

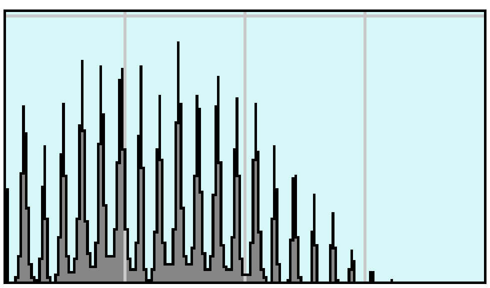


Ratio 1.00 Index 0.01

Ratio 1.00 Index 0.10



Ratio 1.00 Index 1.0



Ratio 1.0 Index 10.

Figure 3. Effect of modulation index

If the modulator and carrier are the same frequency like this, all of the sidebands will be harmonics of that frequency, and the sound will be strongly pitched. You may wonder what happens to the sidebands at frequencies lower than the carrier. If the spacing of the sidebands is the same as the carrier frequency (as it will be if modulator equals carrier), the sideband just below the carrier will be zero in frequency. The sideband below that will be the carrier frequency, but negative. When that concept is applied in reality, the result is the carrier frequency, but 180° out of phase. That sideband therefore weakens or strengthens the fundamental, depending on the modulation index. Further low sidebands interact with upper sidebands in the same way. The regularity of the sidebands produces the strongly harmonic sound usually associated with synthesizers, but if the modulation index is changed during the note (dynamic modulation) the intensity of the sidebands will change in some very voice-like effects.

Figure 4 shows spectra where the carrier and modulator are different frequencies.

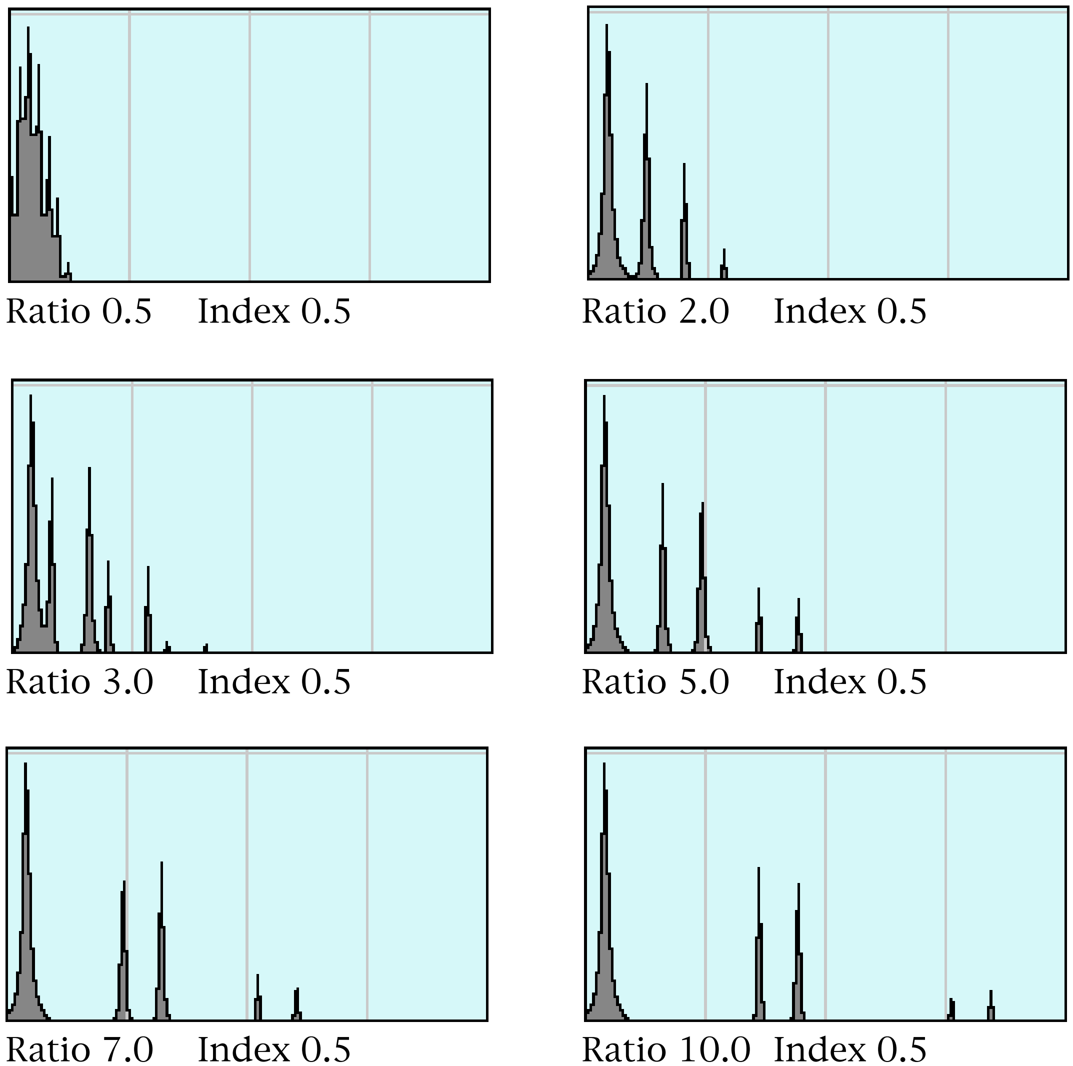
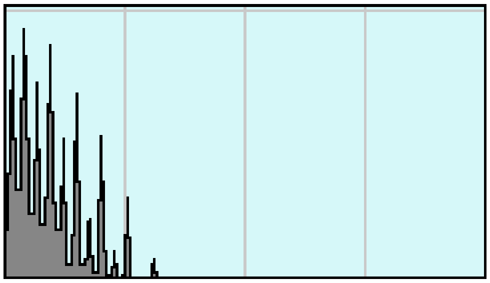


Figure 4. Rational frequency modulation.

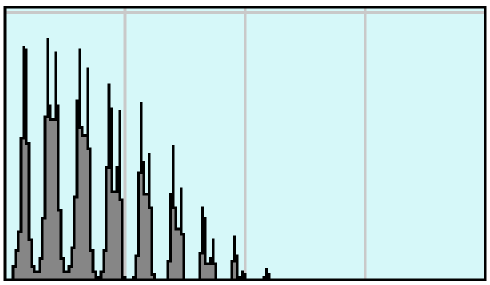
If the frequencies of the carrier and modulator are different but rationally related, the result will again be strongly harmonic, and the pitch will be the root of the implied series. (For instance, frequencies of 400hz and 500hz imply a root of 100hz. This could be produced by a modulator ratio of 2.25 applied to a carrier of 400 hz.) If the carrier is the higher frequency[2], the resultant sound will be quite bright, sounding like a high pass effect at low modulation and becoming very brash as the modulation increases. The frequency of the carrier is always prominent. If the carrier is the lower frequency, the sound will have "missing" harmonics, and those that are present will appear in pairs. At low modulation index, you will hear two distinct pitches in the tone; as the index is increased, the timbre of the upper pitch seems to become brighter.

[2] Produced by a ratio of < 1.0



a. Ratio 1.33 Index 1.0

b. Ratio 0.71 Index 1.0

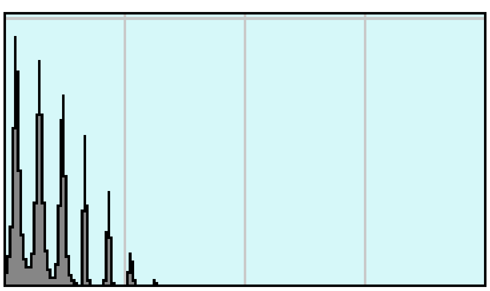


c. Ratio 2.718281 Index 2.4

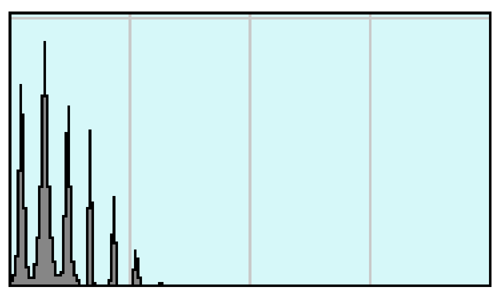
d. Ratio 0.36787 Index 2.4

Figure 4. Irrational Modulation Ratios

If the frequencies of the carrier and modulator are not rationally related, the tone will have a less definite pitch, and will have a rich sound. Very often the effect is of two tones, a weak pure tone at the carrier frequency, plus a rough sound with a vague pitch. With careful adjustment of the modulation index, the carrier tone can be nearly eliminated. Many of these sounds are bell like.



a. Ratio 1.999 Index 1.0

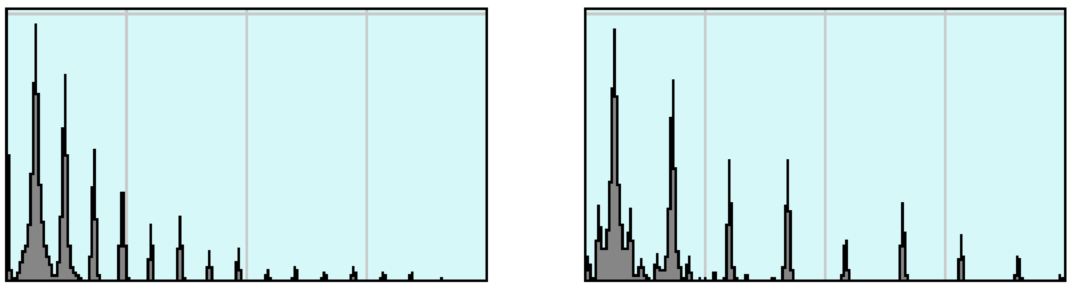


b. Ratio 1.999 Index 1.0

Figure 5. Nearly Rational Modulation Ratios

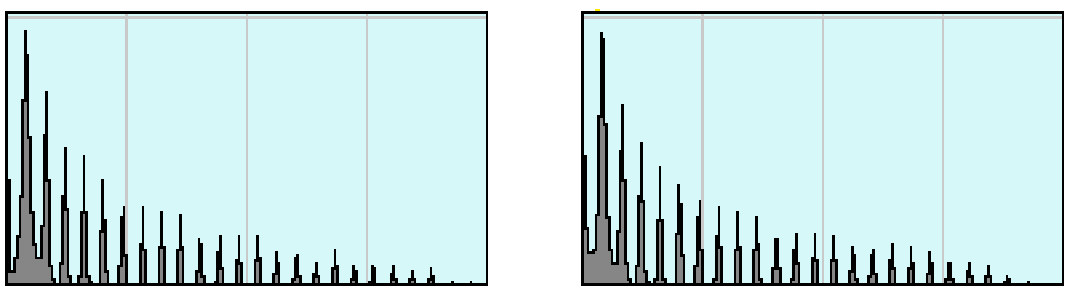
If the frequencies of the carrier and modulator are close to harmonic, **timbral beating** will occur. This is illustrated in figures 5a and 5b. In this example the carrier frequency is 523 Hz. The modulator is 1.999 times this, or 1045 hz. The first lower sideband (reflected through zero) falls at 522 hz. The resulting sound will change from the spectrum of 3e to that of 3f once a second. This gives a warm vibrato.

Very rich spectra can be produced by using complex waveforms for the modulator instead of sine waves. If you choose tri~, rect~ or saw~ for the modulator waveform, you can get results as shown in figure 6 and 7. Note that with saw~ and rect~ (with a duty cycle other than 0.5) there will be some pitch shift as the modulation is applied.



Ratio 1.0 Index 1.0 Ratio 2.0 Index 1.0

Figure 6. Modulation by Tri~ Waveforms



Ratio 1.0 Index 0.5 Ratio 2.0 Index 1.0

Figure 7. Modulation by Rect~ and Saw~ Waveforms

You can also get complex modulation waveforms by modulating the modulators, which I'll demonstrate below.

Envelopes

The FM patch shown above applies a simple envelope to the modulation as well as the amplitude of the note, and can hear a change in the sound as the note fades- that's due to a reduction in the modulation index. Figure 8 shows sonograms of the sound with different decay rates. You can see the upper components fade as the modulation envelope closes.

Decay = 500

Decay = 1000

Figure 8. Effect of envelope on modulation

In a sonogram, each component appears as a horizontal line- its darkness indicates amplitude. Figure 8 shows the upper partials decaying before the lower. If two modulators are used, with different envelopes, there will be a change of timbre during the note, as figure 9 indicates.

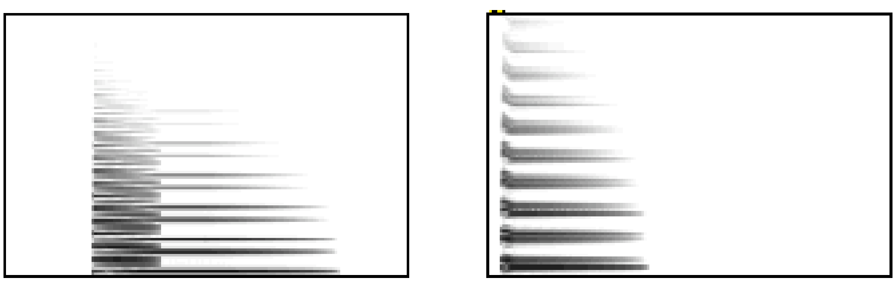


Figure 9. Effect of two modulators

Practical FM patchers

Figure 10. shows a playable FM mono instrument developed from figure 1. I have simplified the look with some encapsulation, which made it straightforward to add a second modulator.

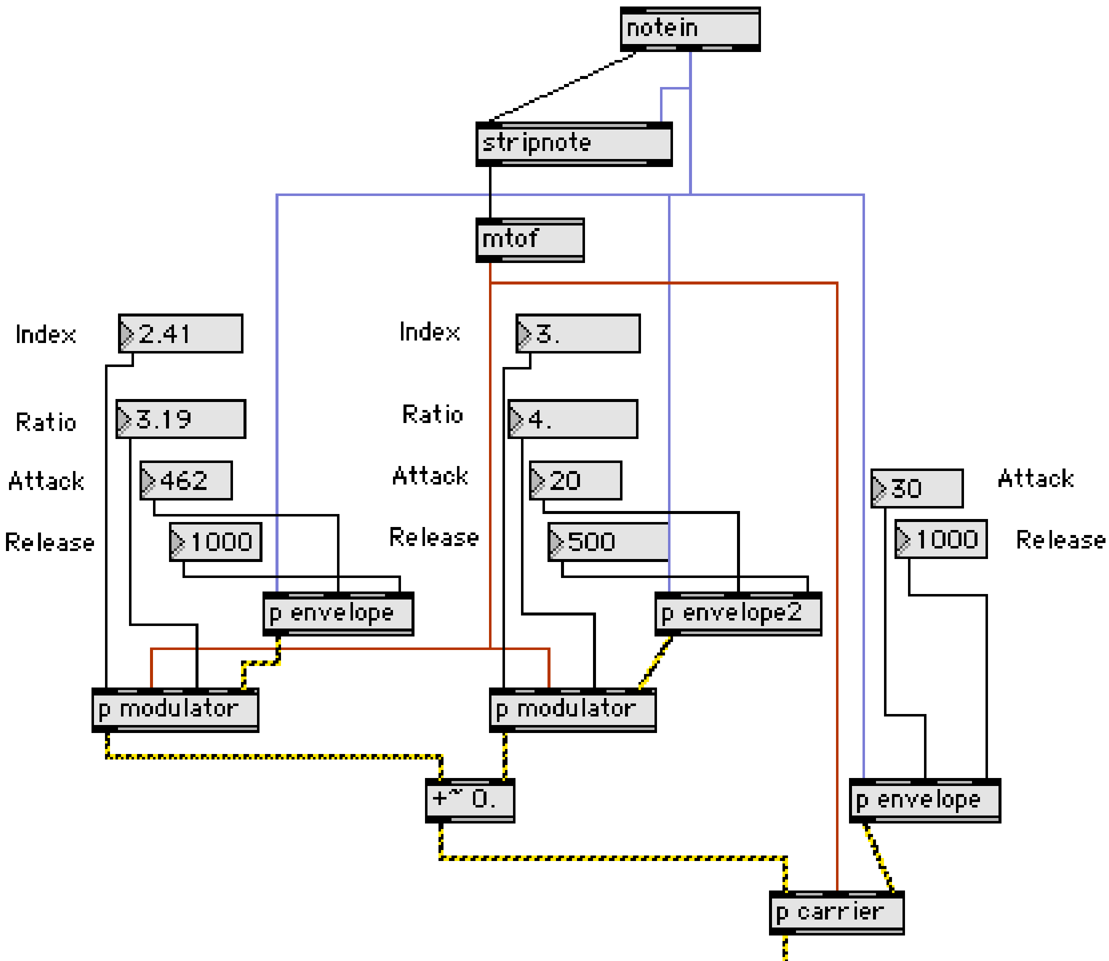
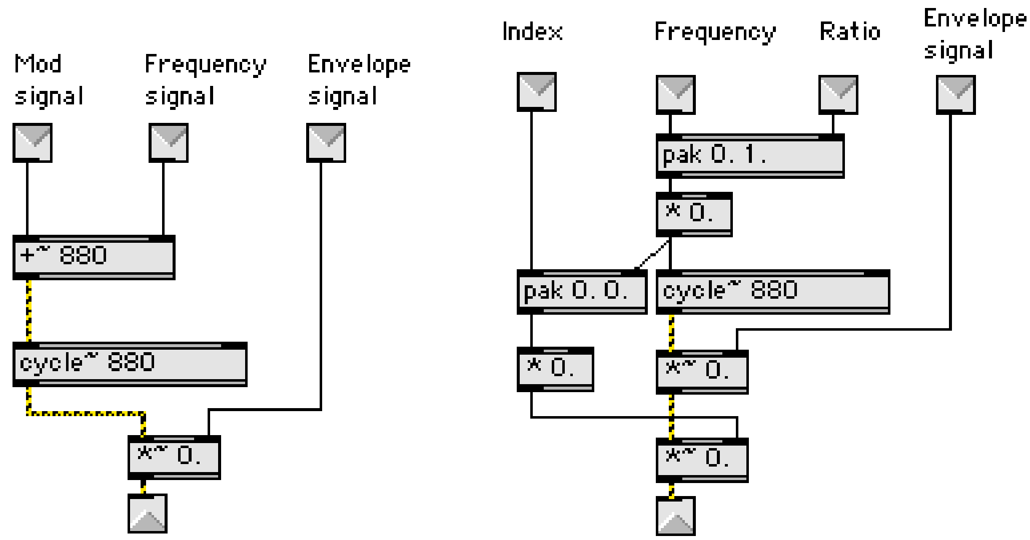


Figure 10. An FM instrument
The note in velocity is sent to three envelope subpatches as well as stripnote. The note number is converted to frequency, which is sent to the modulators and carrier subpatches. Index, ratio and time parameters are set with number boxes, controls will be added later. Figure 11 shows the content of the oscillator subpatchers.



Carrier Modulator

Figure 11. Subpatches of Fig 10.

The carrier is very simple, just including the adder for a second frequency input and a multiply for amplitude control. The modulator also has the math for ratio and deviation. In the DX7, these modules were called operators. They could be combined in many ways for complex effects.

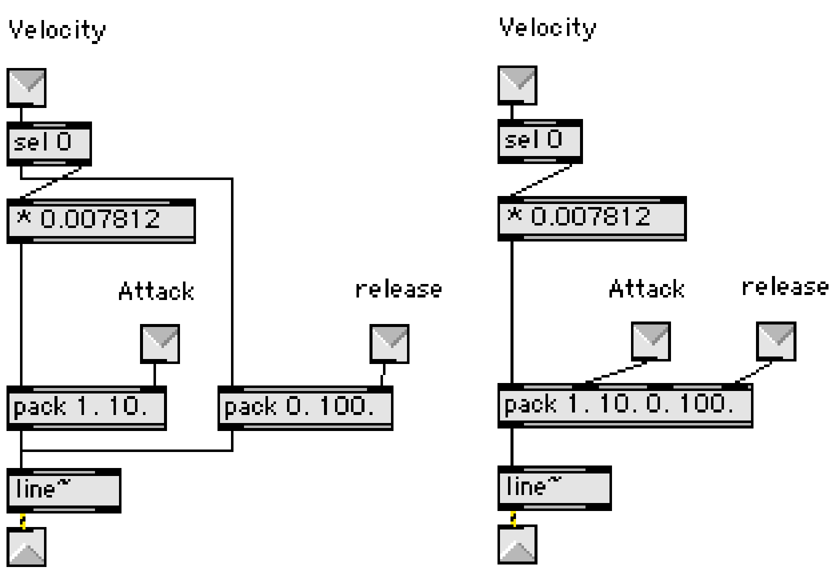


Figure 12. envelopes for fig 10.

The envelopes in figure 10 are equally simple.

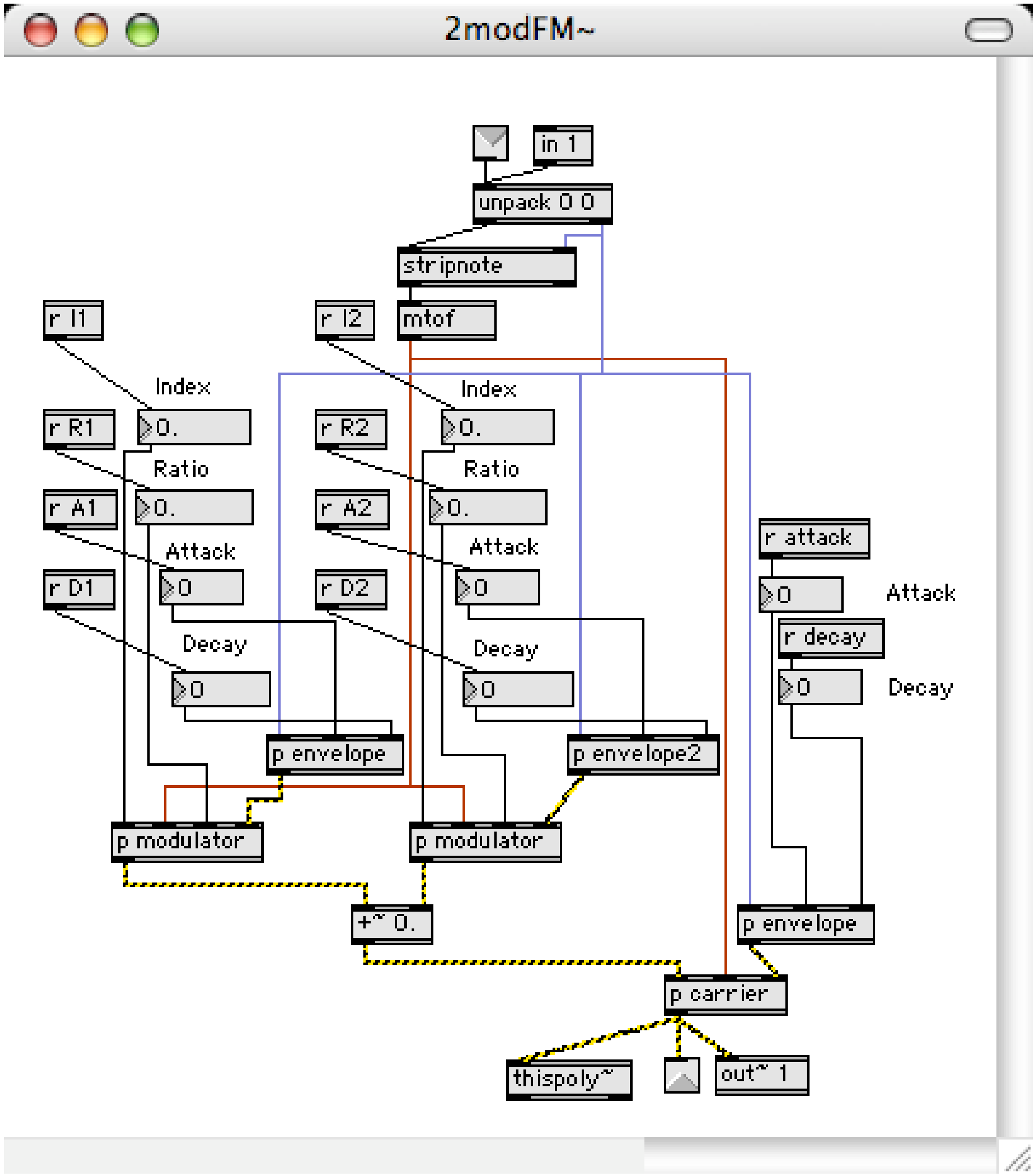


Figure 13. FM subpatch~

In figure 13, the patch has been modified to work inside a poly. This conversion is usually pretty straightforward. Convert notein to unpack, add an inlet for the note info (as a list of pitch, velocity) and an outlet for the signal. Thispoly~ and some receive objects for the parameters complete the instrument. The master patch is then quite simple, as in figure 14.

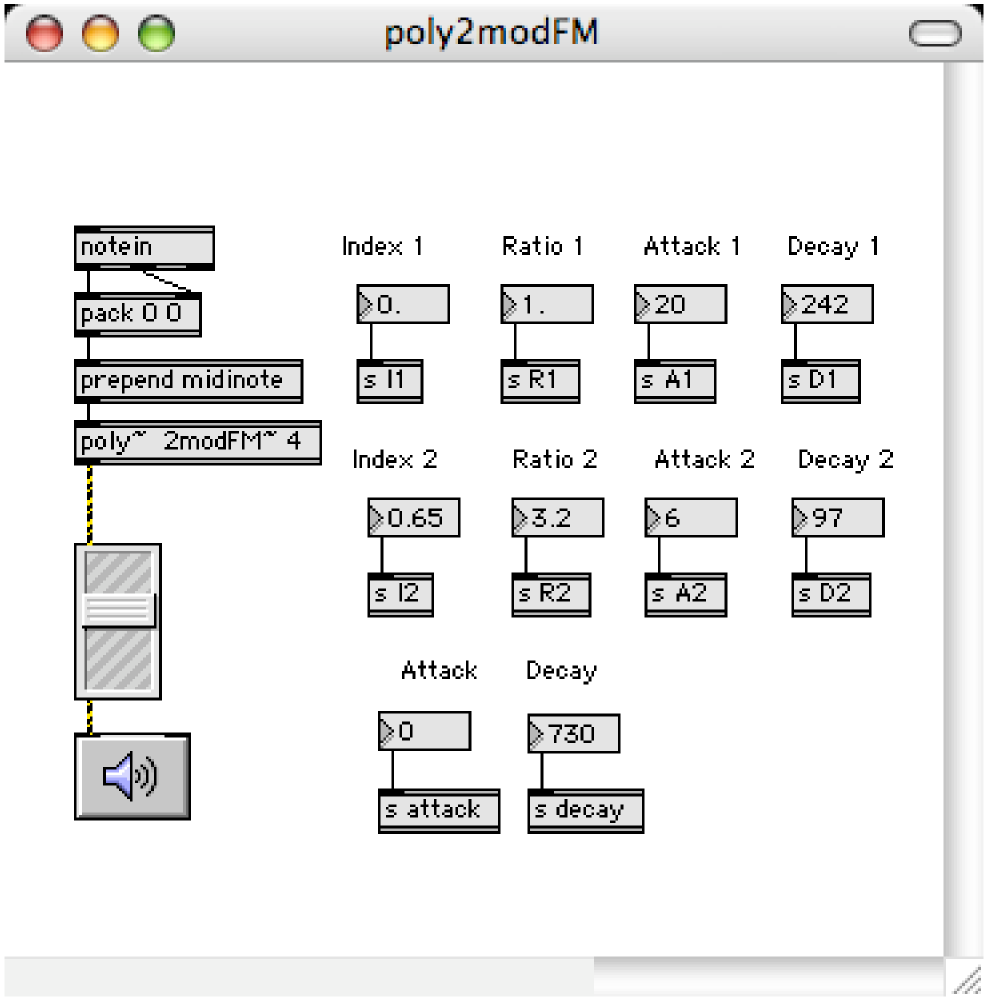


Figure 14. A complete FM instrument

Ideas for further investigation.

The more oscillators you add to a patch, the more possibilities there are for experimentation. Try
Modulating modulators.
Modulators with fixed frequency.
Applying envelopes to modulator frequency.
Adding FM waves with parallel modulator/carrier chains.

John Chowning and David Bristow (who programmed the presets for the DX7) wrote a book called FM Theory and Applications. It's out of print, but if you have a DX instrument, it worth finding a used copy